**Slipchenko M.**
Ph.D., Associate Professor

***Kharkiv State Vocational and PedagogicaL College named after V.I. Vernadskyi***

**Stadnik M.**
Doctor of Technical Sciences, Professor

**Solona O.**
Ph.D., Associate Professor

***Vinnytsia National Agrarian University***

**Сліпченко М.В.**
к.т.н., доцент

***Харківський державний професійно-педагогічний фаховий коледж імені В.І. Вернадського***

**Стаднік М.І.**
д.т.н., професор

**Солона О.В.**
к.т.н., доцент

***Вінницький національний аграрний університет***

***УДК 631.362.36; 621.928.9***
***DOI: 10.37128/2306-8744-2024-4-13***

# FORMULATION OF BOUNDARY CONDITIONS AND EQUATIONS OF DYNAMICS OF GRAIN MIXTURE ON THE PLATE SPREADER OF VIBROCENTRIFUGAL SEPARATOR

*The article investigates the dynamics of grain mixture movement on a plate spreader, taking into account the surface features of the working body and the influence of aerodynamic forces on the cleaning process. The boundary conditions are formulated and equations of particle motion are obtained, which allow us to estimate their trajectory and cleaning efficiency under the influence of air flow. The main factors that affect the uniformity of mixture distribution and quality of its cleaning, in particular the geometric parameters of the spreader, air flow rate, and physical and mechanical characteristics of the particles, are determined.*

*Particular attention is paid to the role of vibration, which can be used to improve the processes of cleaning and pre-mixing the grain mixture before it is fed to spreader. Vibration contributes to uniform distribution of particles in size and density, which, in turn, can increase efficiency of subsequent cleaning by optimizing aerodynamic conditions. The possibility of integrating vibration oscillations into the grain distribution system to ensure flow stability and reduce the unevenness of its movement on surface of spreader is considered.*

*The combination of aerodynamic and vibration effects can provide more efficient removal of impurities, improve the homogeneity of mixture and stabilize its movement during subsequent processing stages. The results obtained can be used to develop improved technological solutions in the field of post-harvest grain processing, in particular to increase the efficiency of cleaning grain mixtures. The proposed approaches are of practical importance for improving the design of equipment used in agricultural production.*

***Keywords:*** *grain mixture, separation, vibration, amplitude, frequency, bulk solids, efficiency, plate spreader, mixing.*

**Introduction.** The efficient separation of granular materials is a crucial aspect of agricultural processing, directly impacting the quality and purity of grain mixtures. Conventional separation methods often struggle with issues such as incomplete impurity removal, high energy consumption, and material loss. Vibrational technologies have emerged as a promising solution, significantly improving the efficiency of separation by enhancing particle mobility and stratification.

This study focuses on the theoretical and experimental investigation of vibrational separation applied to grain and feed mixtures. The research explores how vibrational energy influences the movement, stratification, and sorting of particles, thereby optimizing the removal of impurities. By analyzing the specific boundary conditions and dynamic equations governing particle motion on a vibratory separator, a comprehensive evaluation of key operational parameters is conducted [1].

Additionally, the study examines the potential integration of vibrational separation with air-assisted cleaning techniques to further enhance the effectiveness of the process. The combination of controlled airflow and vibrational energy facilitates a more precise and efficient separation, reducing material losses and improving overall product quality. While vibrational mixing remains a relevant aspect of material processing, its role in ensuring uniform distribution before or after separation is considered as a complementary factor [1, 2].

The outcomes aim to advance grain cleaning operations by refining separation efficiency and setting the stage for further innovations in agricultural material processing.

**Formulation of the problem**. One of the directions of increasing the efficiency of cleaning grain mixtures (GM) from light impurities by vibrocentrifugal separators of JSC «VibroSeparator» (Zhitomir) is the improvement of air cleaning. For this purpose, a new fan-ring cone-cascade pneumatic separating device with air-permeable cone-cascade surface has been developed [1-3]. Compilation of the equations of dynamics of the cleaned air cleaner requires determination of boundary conditions.

**The aim of the research**. Determination of boundary conditions and formulation of the equations of dynamics of GM on the plate spreader (PS) of the vibrating centrifugal separator.

**Results of the research**. GM with impurities slides along the solid wall PS. Then the kinematic condition of non-penetration of the mixture through this wall $S_2$ is fulfilled. [4]:

$$\vec{v}\cdot\vec{n}=0 \quad (S_2). \tag{1}$$

In addition, it is necessary to take into account the action of friction forces on the PS wall. We will consider the so-called «fast movements» of the bulk medium [5]. At such motion the medium experiences sliding friction resistance [6]. In this case, there is a tangential component of the friction force, satisfying Coulomb's law of dry friction:

$$T=f_e N, \tag{2}$$

In terms of continuum mechanics, formula (2) can be written through the stress tensor [4]. The stresses $\vec{p}$ on the surface $S_2$ are related to the stress tensor by the Cauchy relation:

$$\begin{gathered}\vec{p}=p^j\vec{E}_j(r,\varphi,0)=p^\alpha\vec{e}_\alpha+p^3\vec{n}=\\ =\vec{n}\cdot\hat{\sigma}=n_i\sigma^{ij}\vec{E}_j(r,\varphi,0)=\sigma^{3\alpha}\vec{e}_\alpha+\sigma^{33}\vec{n},\\ (n=0),\end{gathered} \tag{3}$$

where summation by repeating Latin indices is from 1 to 3, and by Greek indices from 1 to 2. In expansions of the stress vector by the basis of the curvilinear coordinate system, the first summand $p^\alpha\vec{e}_\alpha\left(T=\left|p^\alpha\vec{e}_\alpha\right|\right)$ corresponds to the tangential component, the second $p^3\vec{n}\left(N=\left|p^3\right|\right)$ – normal. Then from (3) follows the equality:

$$\left(\sigma^{3\alpha}\vec{e}_\alpha\cdot\sigma^{3\beta}\vec{e}_\beta\right)^{1/2}=\left(a_{\alpha\beta}\sigma^{3\alpha}\sigma^{3\beta}\right)^{1/2}=f_e\left|\sigma^{33}\right|, \tag{4}$$

which, taking into account a particular type of surface $S_2$, can be written in the form:

$$\left[\left(1+Z'^2\right)\left(\sigma^{31}\right)^2+r^2\left(\sigma^{32}\right)^2\right]^{1/2}=f_e\left|\sigma^{33}\right|, \quad (n=0). \tag{5}$$

Let us draw up the equilibrium conditions on the free surface of the GM (Fig. 1). In the curvilinear coordinate system ($r$, $\varphi$, $n$) surface $S_1$ define by equation:

$$h(t,r,\varphi)-n=0. \tag{6}$$

Time differentiation of this relation leads to the equation:

$$\frac{\partial h}{\partial t}+\frac{dr}{dt}\frac{\partial h}{\partial r}+\frac{d\varphi}{dt}\frac{\partial h}{\partial\varphi}-\frac{dn}{dt}\equiv\frac{\partial h}{\partial t}+V^1\frac{\partial h}{\partial r}+V^2\frac{\partial h}{\partial\varphi}-V^3=0. \tag{7}$$

Equation (7) represents the kinematic condition on the free surface $S_1$. This condition contains the contravariant components of the velocity vector $\vec{V}=V^i(r,\varphi)\vec{E}_i(r,\varphi,h(t,r,\varphi))$ surface points $S_1$. To uniquely define the surface $S_1$ it is enough to know the normal component of this velocity. Hereinafter we assume $\vec{V}=\vec{n}_1 V$, then $V^1=V^2=0$, and the boundary condition (7) is simplified and takes the form:

$$\frac{\partial h}{\partial t}-V=0. \tag{8}$$

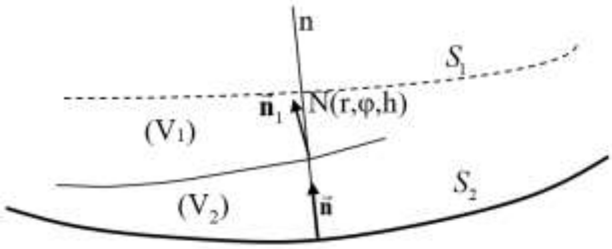


***Fig. 1. To drawing up boundary conditions***

Another group of boundary conditions is given by the equations of continuum dynamics. These are the so-called dynamic conditions [4,7]:

$$\begin{gathered}\left\langle\rho\left(v_n-V\right)\right\rangle=0, \quad (n=h),\\ \left\langle\rho\left(\vec{v}-\vec{V}\right)\left(v_n-V\right)-\vec{n}_1\cdot\hat{\sigma}\right\rangle=0.\end{gathered} \tag{9}$$

Angle brackets denote a jump of the corresponding function at transition from area $V_2$ to area $V_1$ through the interface $S_1=\left(\langle f\rangle=f_1-f_2\right)$.

The GM flux falling on surface $S_1$ in direction of gravity $\vec{g}$, is a set of weakly interacting particles. Therefore, the stress tensor for such a medium is identically equal to zero $\hat{\sigma}_1\equiv 0$. Let $\rho_1,\vec{V}_1\equiv 0$ - and velocities of the incident mixture flow. Then (9) can be represented in the form:

$$\begin{gathered}\rho_1 v_{1n}-\rho_2 v_{2n}-\left(\rho_1-\rho_2\right)V=0, \quad (n=h),\\ \rho_1\left(\vec{v}_1-\vec{n}_1 V\right)\left(v_{1n}-V\right)-\rho_2\left(\vec{v}_2-\vec{n}_1 V\right)\left(v_{1n}-V\right)+\vec{n}_1\cdot\hat{\sigma}=0.\end{gathered} \tag{10}$$

Let us write the vector equation of the surface $S_1$:

$$\vec{r}=\vec{R}_1(r,\varphi)=\vec{R}_0(r,\varphi)+h(t,r,\varphi)\vec{n}(r,\varphi). \tag{11}$$

Basis vectors $\vec{T}_\alpha(\alpha=1{,}2)$ surface $S_1$ are defined by the expressions:

$$\vec{T}_1=\frac{\partial\vec{r}}{\partial r}=\frac{\partial\vec{R}_0}{\partial r}+\frac{\partial h}{\partial r}\vec{n}+h\frac{\partial\vec{n}}{\partial r}=\vec{e}_1+h\frac{\partial\vec{n}}{\partial r}+\frac{\partial h}{\partial r}\vec{n}; \quad (12)$$

$$\vec{T}_2=\frac{\partial\vec{r}}{\partial\varphi}=\frac{\partial\vec{R}_0}{\partial\varphi}+\frac{\partial h}{\partial\varphi}\vec{n}+h\frac{\partial\vec{n}}{\partial\varphi}=\vec{e}_2+h\frac{\partial\vec{n}}{\partial\varphi}+\frac{\partial h}{\partial\varphi}\vec{n},$$

and the unit normal vector by the relation:

$$\vec{n}_1=\frac{\vec{T}_1\times\vec{T}_2}{\left|\vec{T}_1\times\vec{T}_2\right|}. \quad (13)$$

From these relations we see that the vectors $\vec{T}_1,\vec{T}_2,\vec{n}$ with accuracy up to small first order $h$ coincide with vectors $\vec{E}_1(r,\varphi,0),\vec{E}_2(r,\varphi,0)$.

Let us write (10) with accuracy up to small first order by $h$:

$$\rho_1 \text{v}_1^3-\rho_2 \text{v}_2^3-(\rho_1-\rho_2)V=0, \qquad (n=h),$$

$$\rho_1\left(\text{v}_1^3-V\right)^2-\rho_2\left(\text{v}_2^3-V\right)^2+\sigma^{33}=0, \quad (14)$$

$$\rho_1\left(\text{v}_1^3-V\right)\text{v}_1^1-\rho_2\left(\text{v}_2^3-V\right)\text{v}_2^1+\sigma^{31}=0,$$

$$\rho_1\left(\text{v}_1^3-V\right)\text{v}_1^2-\rho_2\left(\text{v}_2^3-V\right)\text{v}_2^2+\sigma^{32}=0.$$

Let's consider the area $V_2$ (Fig. 2) currents GM on PS: $d_0/2\le r\le D_1/2$, $0\le\varphi\le\beta_1$, $0\le n\le h(t, r, \varphi)$. Let's denote by $S_p$ section of the specified area by a coordinate surface $r$ = const = $r_p$. The vector assignment of this surface has the form:

$$\vec{r}_p=\vec{r}_p(\varphi,n)=\left(r_p\cos\varphi, r_p\sin\varphi, n\right) \quad (15)$$

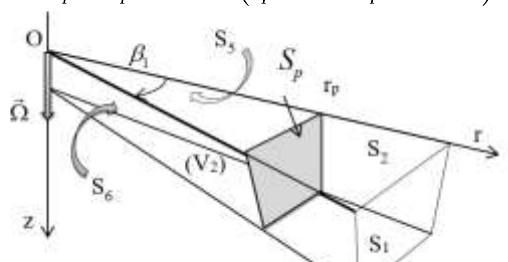


***Fig. 2. Toward a definition of area $V_2$***

The area element of this surface is defined in the form [8,9]:

$$dS=\left|d_{(1)}\vec{r}_p\times d_{(2)}\vec{r}_p\right|=\left|\frac{\partial\vec{r}}{\partial\varphi}\times\frac{\partial\vec{r}}{\partial n}\right|d\varphi\, dn=$$

$$=\left|\begin{array}{l}\left[\left(-r\sin\varphi+\frac{nZ'\sin\varphi}{\sqrt{(Z'^2+1)}}\right)\vec{e}_x+\left(r\cos\varphi-\frac{nZ'\cos\varphi}{\sqrt{(Z'^2+1)}}\right)\vec{e}_{yz}\right]\times\\ \times\left[\left(-\frac{Z'\cos\varphi}{\sqrt{(Z'^2+1)}}\right)\vec{e}_x+\left(-\frac{Z'\sin\varphi}{\sqrt{(Z'^2+1)}}\right)\vec{e}_y+\left(\frac{1}{\sqrt{(Z'^2+1)}}\right)\vec{e}_z\right]\end{array}\right|.$$

With small-order accuracy $\sim o(n)$ the last expression takes the form:

$$dS=r.$$

Let us introduce a non-inertial moving coordinate system $K'$, rigidly connected to the rotating PS with constant angular velocity $\vec{\Omega}$ (fig. 2) PS. Absolute speed $\vec{V}^0$ and absolute acceleration $\vec{w}^0$ particles of a continuous medium according to the theorems of addition of velocities and accelerations are a vector sum of transport velocities $\vec{V}^e$ and accelerations $\vec{w}^e$, relative speeds $\vec{V}$ and accelerations $\vec{w}$, Coriolis acceleration $\vec{w}^c$ [10, 11]:

$$\vec{\text{v}}^0=\vec{\text{v}}^e+\vec{\text{v}};$$

$$\vec{w}^0=\vec{w}^e+\vec{w}^c+\vec{w}=\vec{\Omega}\times\left(\vec{\Omega}\times\vec{r}\right)+2\vec{\Omega}\times\vec{\text{v}}+\vec{w}= \quad (16)$$

$$=\vec{\Omega}\left(\vec{\Omega}\cdot\vec{r}\right)-\Omega^2\vec{r}+2\vec{\Omega}\times\vec{\text{v}}+\vec{w}.$$

Here are the vector and scalar product of vectors $\vec{\Omega},\vec{r}$, which should be written in the curvilinear coordinate system ($r$, $\varphi$, $n$):

$$\vec{\Omega}=\Omega\vec{e}_z;$$

$$\vec{r}=\vec{R}_0+n\vec{n}=$$

$$=\left[r\cos\varphi-\frac{nZ'\cos\varphi}{\sqrt{(Z'^2+1)}}\right]\vec{e}_x+\left[r\sin\varphi-\frac{nZ'\sin\varphi}{\sqrt{(Z'^2+1)}}\right]\vec{e}_y+\left[Z+\frac{n}{\sqrt{(Z'^2+1)}}\right]\vec{e}_z.$$

The equations of GM dynamics expressing the law of conservation of mass [4] taking into account specific values of TR geometry take the following form

$$\frac{\partial\varepsilon_\alpha}{\partial t}+\frac{1}{r\sqrt{(Z'^2+1)}}\frac{\partial}{\partial r}\left(r\sqrt{(Z'^2+1)}\varepsilon_\alpha\left(\vec{\text{v}}_\alpha+\vec{\Omega}\times\vec{r}\right)^1\right)+$$

$$+\frac{\partial\varepsilon_\alpha\left(\vec{\text{v}}_\alpha+\vec{\Omega}\times\vec{r}\right)^2}{\partial\varphi}+\frac{\partial\varepsilon_\alpha\left(\vec{\text{v}}_\alpha+\vec{\Omega}\times\vec{r}\right)^3}{\partial n}=0, \quad (17)$$

$$(\alpha=1,2);$$

$$\frac{\partial\varepsilon_1\vec{\text{v}}_1}{\partial t}=\frac{1}{\rho_1^0}div\left(\hat{\sigma}_1-\rho_1^0\varepsilon_1\vec{\text{v}}_1\vec{\text{v}}_1\right)+\varepsilon_1\vec{g}-$$

$$-\varepsilon_1\left[\vec{\Omega}\left(\vec{\Omega}\cdot\vec{r}\right)-\Omega\vec{r}+2\vec{\Omega}\times\vec{\text{v}}_1\right];$$

$$\frac{\partial\varepsilon_2\vec{\text{v}}_2}{\partial t}=-\frac{1}{\rho_2^0}div\left(\rho_2^0\varepsilon_2\vec{\text{v}}_2\vec{\text{v}}_2\right)+\frac{9\mu_1\varepsilon_2}{2a^2\rho_2^0}\left(\vec{\text{v}}_1-\vec{\text{v}}_2\right)+\varepsilon_2\vec{g}- \quad (18$$

$$-\varepsilon_2\left[\vec{\Omega}\left(\vec{\Omega}\cdot\vec{r}\right)-\Omega\vec{r}+2\vec{\Omega}\times\vec{\text{v}}_2\right].$$

With accuracy up to terms of order ~o(n), the vectors of initial basis [8, 12] are used to construct the vectors of reciprocal basis $\vec{E}_1,\vec{E}_2,\vec{E}_3$:

$$\vec{E}^1=\frac{\vec{E}_2\times\vec{E}_3}{\sqrt{\vec{E}_1\cdot\left(\vec{E}_2\times\vec{E}_3\right)}}=\left(\frac{\cos\varphi}{Z'^2+1},\frac{\sin\varphi}{Z'^2+1},\frac{Z'}{Z'^2+1}\right);$$

$$\vec{E}^2=\frac{\vec{E}_3\times\vec{E}_1}{\sqrt{\vec{E}_1\cdot\left(\vec{E}_2\times\vec{E}_3\right)}}=\left(-\frac{\sin\varphi}{r},\frac{\cos\varphi}{r},0\right); \quad (19)$$

$$\vec{E}^3=\frac{\vec{E}_1\times\vec{E}_2}{\sqrt{\vec{E}_1\cdot\left(\vec{E}_2\times\vec{E}_3\right)}}=\left(-\frac{Z'\cos\varphi}{\sqrt{(Z'^2+1)}},-\frac{Z'\sin\varphi}{\sqrt{(Z'^2+1)}},\frac{1}{\sqrt{(Z'^2+1)}}\right).$$

Then the contravariant components of vectors $\vec{\Omega}$, $\vec{\Omega}\times\vec{r}$ and $\vec{\Omega}\times\vec{v}$ look like:

$$\Omega^1 = \Omega \frac{Z'}{Z'^2+1}, \quad \Omega^2 = 0, \quad \Omega^3 = \Omega \frac{Z'}{\sqrt{Z'^2+1}};$$

$$\left(\vec{\Omega}\times\vec{r}\right)^1 = 0, \quad \left(\vec{\Omega}\times\vec{r}\right)^2 = \Omega, \quad \left(\vec{\Omega}\times\vec{r}\right)^3 = 0; \qquad (20)$$

$$\left(\vec{\Omega}\times\vec{\mathrm{v}}\right)^1 = -\Omega \frac{r\,\mathrm{v}^2}{Z'^2+1}, \quad \left(\vec{\Omega}\times\vec{\mathrm{v}}\right)^2 = \Omega\left[\frac{\mathrm{v}^1}{r} - \frac{\mathrm{v}^3 Z'}{r\sqrt{Z'^2+1}}\right],$$

$$\left(\vec{\Omega}\times\vec{\mathrm{v}}\right)^3 = \Omega \frac{rZ'\,\mathrm{v}^2}{\sqrt{\left(Z'^2+1\right)}},$$

and the scalar product $\vec{\Omega}\times\vec{r}$ equal:

$$\vec{\Omega}\cdot\vec{r} = \Omega Z. \qquad (21)$$

In the following we will consider stationary motion of the medium. In this case, the summands containing time partial derivatives turn to zero. Taking into account all the above-mentioned, (17), (18) are written in the form of:

$$\frac{1}{r\sqrt{\left(Z'^2+1\right)}}\frac{\partial}{\partial r}\left(r\sqrt{\left(Z'^2+1\right)}\varepsilon_\alpha\,\mathrm{v}_\alpha^1\right) + \frac{\partial\varepsilon_\alpha\left(\mathrm{v}_\alpha^2+\Omega\right)}{\partial\varphi} + \frac{\partial\varepsilon_\alpha\,\mathrm{v}_\alpha^3}{\partial n} = 0, \qquad (22)$$

$$(\alpha = 1,2);$$

$$\frac{1}{\rho_1^0}\left[\frac{\partial}{\partial r}\left(\sigma^{11}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^1\,\mathrm{v}_1^1\right)+\frac{\partial}{\partial\varphi}\left(\sigma^{21}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^2\,\mathrm{v}_1^1\right)+\frac{\partial}{\partial n}\left(\sigma^{13}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^3\,\mathrm{v}_1^1\right)+\right.$$

$$\left.+A_{11}\left(\sigma^{11}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^1\,\mathrm{v}_1^1\right)-A_{13}\left(\sigma^{13}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^3\,\mathrm{v}_1^1\right)-A_{22}\left(\sigma^{22}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^2\,\mathrm{v}_1^2\right)\right]+$$

$$+\varepsilon_1 g\frac{Z'}{Z'^2+1}-\varepsilon_1\left[\Omega^2\frac{ZZ'}{Z'^2+1}-\Omega^2 r\cos\varphi-2\Omega\frac{r\,\mathrm{v}_1^2}{Z'^2+1}\right]=0;$$

$$\frac{1}{\rho_1^0}\left[\frac{\partial}{\partial r}\left(\sigma^{12}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^1\,\mathrm{v}_1^2\right)+\frac{\partial}{\partial\varphi}\left(\sigma^{22}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^2\,\mathrm{v}_1^2\right)+\frac{\partial}{\partial n}\left(\sigma^{23}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^3\,\mathrm{v}_1^2\right)+\right.$$

$$\left.+B_{12}\left(\sigma^{12}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^1\,\mathrm{v}_1^2\right)-B_{23}\left(\sigma^{23}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^3\,\mathrm{v}_1^2\right)\right]-$$

$$-\varepsilon_1\left[-\Omega^2 r\sin\varphi+2\Omega\left(\frac{\mathrm{v}^1}{r}-\frac{Z'}{r\sqrt{Z'^2+1}}\mathrm{v}_1^3\right)\right]=0;$$

$$\frac{1}{\rho_1^0}\left[\frac{\partial}{\partial r}\left(\sigma^{13}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^1\,\mathrm{v}_1^3\right)+\frac{\partial}{\partial\varphi}\left(\sigma^{23}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^2\,\mathrm{v}_1^3\right)+\frac{\partial}{\partial n}\left(\sigma^{33}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^3\,\mathrm{v}_1^3\right)+\right.$$

$$+C_{11}\left(\sigma^{11}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^1\,\mathrm{v}_1^1\right)+C_{13}\left(\sigma^{13}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^1\,\mathrm{v}_1^3\right)+ \qquad (23)$$

$$\left.+C_{22}\left(\sigma^{22}-\rho_1^0\varepsilon_1\,\mathrm{v}_1^2\,\mathrm{v}_1^2\right)-C_{33}\left(\sigma^{33}-\rho_1^0\varepsilon_1\,\mathrm{v}^3\,\mathrm{v}_1^3\right)\right]+$$

$$+\varepsilon_1 g\frac{Z'}{\sqrt{Z'^2+1}}-\varepsilon_1\left[\Omega^2\frac{ZZ'}{Z'^2+1}-\Omega^2 r\cos\varphi-2\Omega\frac{rZ'\,\mathrm{v}_1^2}{\sqrt{Z'^2+1}}\right]=0;$$

$$\frac{1}{\rho_2^0}\left[\frac{\partial}{\partial r}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^1\,\mathrm{v}_2^1\right)+\frac{\partial}{\partial\varphi}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^2\,\mathrm{v}_2^2\right)+\frac{\partial}{\partial n}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^3\,\mathrm{v}_2^1\right)+\right.$$

$$+A_{11}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^1\,\mathrm{v}_2^1\right)\varepsilon_2\,\mathrm{v}_2^1\,\mathrm{v}_2^1-A_{13}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^3\,\mathrm{v}_2^1\right)\varepsilon_2\,\mathrm{v}_2^3\,\mathrm{v}_2^1$$

$$\left.-A_{22}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^2\,\mathrm{v}_2^2\right)\varepsilon_2\,\mathrm{v}_2^2\,\mathrm{v}_2^2\right]+\frac{9\mu_1\varepsilon_2}{2a^2\rho_2^0}\left(\mathrm{v}_1^1-\mathrm{v}_2^1\right)+$$

$$+\varepsilon_2 g\frac{Z'}{Z'^2+1}-\varepsilon_2\left[\Omega^2\frac{ZZ'}{Z'^2+1}-\Omega^2 r\cos\varphi-2\Omega\frac{r\,\mathrm{v}_2^2}{Z'^2+1}\right]=0;$$

$$\frac{1}{\rho_2^0}\left[\frac{\partial}{\partial r}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^1\,\mathrm{v}_2^2\right)+\frac{\partial}{\partial\varphi}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^2\,\mathrm{v}_2^2\right)+\frac{\partial}{\partial n}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^3\,\mathrm{v}_2^2\right)+\right.$$

$$\left.+B_{12}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^1\,\mathrm{v}_2^2\right)-B_{23}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^3\,\mathrm{v}_2^2\right)\right]+ \qquad (24)$$

$$+\frac{9\mu_1\varepsilon_2}{2a^2\rho_2^0}\left(\mathrm{v}_1^2-\mathrm{v}_2^2\right)-\varepsilon_2\left[-\Omega^2 r\sin\varphi+2\Omega\left(\frac{\mathrm{v}_2^1}{r}-\frac{Z'}{r\sqrt{Z'^2+1}}\mathrm{v}_2^3\right)\right]=0;$$

$$\frac{1}{\rho_2^0}\left[\frac{\partial}{\partial r}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^1\,\mathrm{v}_2^3\right)+\frac{\partial}{\partial\varphi}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^2\,\mathrm{v}_2^3\right)+\frac{\partial}{\partial n}\left(-\rho_2^0\varepsilon_2\,\mathrm{v}_2^3\,\mathrm{v}_2^3\right)\right]+$$

$$C_{11}\left(\varepsilon_2\,\mathrm{v}_2^1\,\mathrm{v}_2^1\right)+C_{13}\left(\varepsilon_2\,\mathrm{v}_2^1\,\mathrm{v}_2^3\right)+C_{22}\left(\varepsilon_2\,\mathrm{v}_2^2\,\mathrm{v}_2^2\right)-C_{33}\left(\varepsilon_2\,\mathrm{v}_2^3\,\mathrm{v}_2^3\right)\Big]+$$

$$+\frac{9\mu_1\varepsilon_2}{2a^2\rho_2^0}\left(\mathrm{v}_1^3-\mathrm{v}_2^3\right)+\varepsilon_2 g\frac{Z'}{\sqrt{Z'^2+1}}-\varepsilon_2\left[\Omega^2\frac{ZZ'}{Z'^2+1}-\Omega r\cos\varphi-2\Omega\frac{rZ'\,\mathrm{v}_2^2}{\sqrt{Z'^2+1}}\right]=0.$$

***Conclusions.***

The results of this study confirm that vibrational separation is a highly effective technique for improving the quality and efficiency of grain cleaning processes. The introduction of controlled vibrational energy enhances particle stratification, reduces impurity content, and optimizes material flow, leading to higher precision in separating desired components from unwanted fractions.

Furthermore, the integration of vibrational separation with air-assisted cleaning has demonstrated significant improvements in removing lightweight impurities while minimizing material loss. This hybrid approach enhances the overall efficiency of grain processing operations, making them more sustainable and economically viable.

Boundary conditions and equations of GM dynamics by PS with regard to a specific type of its surface are compiled. The study of the obtained equations is necessary to establish regularities and improve the efficiency of GM cleaning by air flow at its descent from TS.

**References**

1. Slipchenko, M., Bredykhin, V., Kis-Korkishchenko, L., Pak, A., Alfyorov, O. (2023). Construction of a physical-mathematical model of oscillations of the unbalanced vibrator of the pneumatic sorting table. *Eastern-European Journal of Enterprise Technologies*. Vol. 4, no. 7 (124). P. 89-97 [in English].

2. Tishchenko, L.M., Kharchenko, S.O., Slipchenko, M.V. Patent na korysnu model' Ukrayiny. Vibrovidtsentrovyy separator [Patent for a utility model of Ukraine. Vibrocentrifugal separator]. № 50587. 10.06.2010 [in Ukrainian].

3. Bredykhin, V. et al. (2023). Improving Efficiency of Corn Seed Separation and Calibration Process. *Agricultural Engineering*. Vol. 27, no. 1. P. 241-253 [in English].

4. Olshanskyi, V.P., Slipchenko, M.V., Kharchenko, S.O., Polievoda, Yu.A. (2021). Nablyzhenyy sposib rozrakhunku zernopotoku v vertykal'nomu tsylindrychnomu vibroresheti [Approximate method for calculating grain flow in a vertical cylindrical vibrating screen]. *Vibratsiyi v tekhnitsi i tekhnolohiyi – Vibrations in engineering and technology*, 1 (100), P. 57-65 [in Ukrainan].

5. Polievoda Yu.A., Volynets, Ye.O. (2021). Doslidzhennya vibroreolohichnykh modeley sharu sypkoho seredovyshcha [Research of vibro-rheological models of a layer of loose medium]. *Tekhnika, enerhetyka, transport APK – Technology, energy, agriculture transport AIC*, 1 (112), P. 60-69 [in Ukrainian].

6. Olshanskyi, V., Slipchenko, M., Kharchenko, S., Kovalyshyn, S., Barsuk, A. (2021). Pro rukh neodnoridnoyi zernosumishi po vertykal'nomu vibroreshetu [On the movement of a heterogeneous grain mixture along a vertical vibrating screen]. *Inzheneriya pryrodokorystuvannya – Environmental Engineering*, 2 (20), P. 27-31 [in Ukrainian].

7. Olshanskyi, V.P., Bredykhin, V.V., Lukianenko, V.M. (2017). Teoriya separuvannya zernaystuvannya [Grain separation theory]. KhNTUSH. 803 p [in Ukrainian].

8. Solona, O.V., Kupchuk, I.M., Olshanskyi, V.P., Slipchenko, M.V. (2021). Pro analitychnyy rozv'yazok rivnyannya syly udaru dvokh pruzhnykh til [On the analytical solution of the equation of the impact force of two elastic bodies]. *Vibratsiyi v tekhnitsi i tekhnolohiyi – Vibrations in engineering and technology*, 3 (102), P. 34-42 [in Ukrainan].

9. Olkhovskyi, V.O., Dudariev, I.M. (2021). Modelyuvannya rukhu konstruktyvnykh elementiv zernovoho separatora nozhychnoho typu [Modeling the movement of structural elements of a scissor-type grain separator]. *Sil's'kohospodars'ki mashyny – Agricultural machinery*, 46, P. 70-84 [in Ukrainian].

10. Solona, O.V., Kotov, B.I., Spirin, A.V., Kalinichenko R.A. (2016). Obgruntuvannya parametriv poyednanykh protsesiv mikronizatsiyi i podribnennya iz zastosuvannyam vibratsiynykh tekhnolohiy pry pererobtsi zerna na korm [Justification of the parameters of combined micronization and grinding processes using vibration technologies in grain processing for feed]. *Vibratsiyi v tekhnitsi i*

*tekhnolohiyi – Vibrations in engineering and technology*, 3, P. 213-218 [in Ukrainian].

11. Borysiuk, D., Spirin, A., Kupchuk, I., Tverdokhlib, I., Zelinskyi, V., Smyrnov, Ye., Ognevyy, V. (2021). The methodology of determining the place of installation of accelerometers during vibrodiagnostic of controlled axes of wheeled tractors. *Przegląd Elektrotechniczny*, Vol. 97, № 10. P. 44-48 [in English].

12. Spirin, A.V., Tverdokhlib, I.V., Kotov, B.I., Stepanenko, S.P., Shvydya, V.O. (2017). Do pytannya pnevmohravitatsiynoyi separatsiyi zernovykh materialiv [On the issue of pneumogravity separation of grain materials]. *Tekhnika, enerhetyka, transport APK – Technology, energy, agriculture transport AIC*, 4 (99), P. 51-55 [in Ukrainian].

## ФОРМУЛЮВАННЯ ГРАНИЧНИХ УМОВ І РІВНЯНЬ ДИНАМІКИ ЗЕРНОВОЇ СУМІШІ НА ТАРІЛЧАСТОМУ РОЗКИДАЧІ ВІБРОВІДЦЕНТРОВОГО СЕПАРАТОРА

*У статті досліджено динаміку руху зернової суміші на тарілчастому розкидачі, враховуючи особливості поверхні робочого органа та вплив аеродинамічних сил на процес очищення. Сформульовано граничні умови та отримано рівняння руху частинок, що дозволяють оцінити їхню траєкторію та ефективність очищення під дією повітряного потоку. Визначено основні фактори, які впливають на рівномірність розподілу суміші та якість її очистки, зокрема геометричні параметри розкидача, швидкість потоку повітря та фізико-механічні характеристики частинок.*

*Окрему увагу приділено ролі вібраційного впливу, який може використовуватися для покращення процесів очищення та попереднього змішування зернової суміші перед її подачею на розкидач. Вібрація сприяє рівномірному розподілу частинок за розміром та густиною, що, у свою чергу, може підвищити ефективність подальшого очищення за рахунок оптимізації аеродинамічних умов. Розглянуто можливість інтеграції вібраційних коливань у систему розподілу зерна для забезпечення стабільності потоку та зниження нерівномірності його руху на поверхні розкидача.*

*Поєднання аеродинамічного і вібраційного впливу може забезпечити більш ефективне видалення домішок, покращення однорідності суміші та стабілізацію її руху на подальших етапах обробки. Отримані результати можуть бути використані для розробки вдосконалених технологічних рішень у галузі післязбиральної обробки зерна, зокрема для підвищення ефективності очищення зернових сумішей. Запропоновані підходи мають практичне значення для вдосконалення конструкцій обладнання, що застосовується в сільськогосподарському виробництві.*

***Ключові слова:*** *зернова суміш, сепарація, вібрація, амплітуда, частота, сипке середовище, ефективність, тарілчастий розкидач, змішування.*

### *Відомості про авторів*

**Slipchenko Maksym** – Ph.D., Associate Professor, Associate Professor, head of the department of professional education of Kharkiv State Vocational and Pedagogical College named after V.I. Vernadskyi (Kharkov, Ukraine, 61002, email: Slipchenko_M@ukr.net, https://orcid.org/0000-0002-9728-661X).

**Stadnik Mykola** – Doctor of Technical Sciences, Professor of the Department of Electrical Power Engineering, Electrical Engineering and Electromechanics of Vinnytsia National Agrarian University (3 Sonyachna St., Vinnytsia, 21008, Ukraine, e-mail: stadnik1948@gmail.com, https://orcid.org/0000-0003-3895-9607).

**Solona Olena** – Ph.D., Associate Professor, Dean of the Faculty of Production Technology, Processing and Robotics in Livestock Breeding of the Vinnytsia National Agrarian University (3, Sonyachna St., Vinnytsia, Ukraine, 21008, e-mail: solona_o_v@ukr.net, https://orcid.org/0000-0002-4596-0449).

**Сліпченко Максим Володимирович** – кандидат технічних наук, доцент, завідувач відділення професійної освіти Харківського державного професійно-педагогічного фахового коледжу імені В.І. Вернадського (м. Харків, Україна, 61002, email: Slipchenko_M@ukr.net, https://orcid.org/0000-0002-9728-661X).

**Стаднік Микола Іванович** – доктор технічних наук, професор кафедри електроенергетики, електротехніки та електромеханіки Вінницького національного аграрного університету (вул. Сонячна, 3, м. Вінниця, 21008, Україна, e-mail: stadnik1948@gmail.com, https://orcid.org/0000-0003-3895-9607).

**Солона Олена Василівна** – кандидат технічних наук, доцент, декан факультету технології виробництва, переробки та робототехніки у тваринництві Вінницького національного аграрного університету (вул. Сонячна, 3, м. Вінниця, Україна, 21008, e-mail: solona_o_v@ukr.net, https://orcid.org/0000-0002-4596-0449).