**Yaroshenko L.**

Ph.D. in Engineering, Associate Professor

**Slobodianyk A.**

Ph.D. in Engineering, Associate Professor

*Vinnytsia National Agrarian University***Ярошенко Л.В.**

К.Т.Н., доцент

**Слободяник А.Д.**

К.Т.Н., доцент

*Вінницький національний аграрний університет***УДК 621.9.048.6****DOI: 10.37128/2306-8744-2025-3-5**

## ELASTIC SUSPENSIONS OF VIBRATING MACHINES WITH ADJUSTABLE ELECTROMECHANICAL DRIVE

High requirements for reliability, durability, power and productivity of vibrating machines are directly determined by the quality of their elastic elements, in particular their endurance and resistance to the action of intense cyclic loads. For effective vibration isolation of the working body of a resonant vibrating machine from the frame, the suspensions must have sufficient softness, while increasing productivity requires the maximum possible rigidity of these elements.

The paper provides recommendations for the calculation and design of elastic suspensions of vibrating technological machines operating in both sub-resonant and resonant modes, as well as elastic elements with adjustable stiffness for machines with an adjustable electromechanical drive. Cylindrical helical compression springs are advisable to use in relatively small vibration installations with a container volume of up to 100 dm<sup>3</sup>, installing them without tilting. For more powerful machines, it is recommended to use combined suspensions or rubber-cord pneumatic cylinders placed at an angle to the longitudinal and transverse planes.

Of particular importance are resonant-type vibrating machines, which are distinguished by a number of technological, energy and operational advantages, but at the same time are characterized by the dependence of dynamic parameters on the degree of loading.

In view of this, a promising direction is the use of suspensions with adjustable stiffness, which ensure the stability of the dynamic mode of operation of the vibrating machine and allow it to be changed in accordance with a given algorithm. An effective solution is also combined elastic suspensions, which combine a diaphragm pneumatic drive with a rubber-metal block hinge. The torsional stiffness of such systems can be changed by adjusting the number of block hinges, and the stiffness of the diaphragm by changing the gas pressure in the suspension cavity.

**Keywords:** elastic suspensions, vibration machine, adjustable electromechanical drive, stiffness, natural frequency of oscillations, resonant mode, pneumatic cylinder.

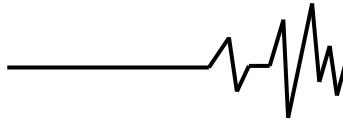
**Problem statement.** High requirements for reliability, durability, power and productivity of vibrating machines directly depend on the quality of their elastic elements, in particular on their endurance and reliability under the influence of intense cyclic loads.

Elastic suspensions of such machines simultaneously perform two main functions - they perceive the mass of vibrating parts and provide vibration isolation of the frame, therefore they must combine elasticity and sufficient stability, because

they are often the only connecting element between the working units and the base.

Given the technological features, most vibrating machines operate in a non-resonant mode, so the overall stiffness of the suspension must ensure that the natural frequency of oscillations of the working bodies is 3–4 times lower than the frequency of the disturbing force or the cyclic frequency of rotation of the unbalanced shaft of the vibration drive [1-4].

However, more economical is the resonant mode of operation, when the frequency of natural



(free) oscillations of the working bodies is equal to the cyclic frequency of oscillations of the disturbing force. During resonance, there is an effective accumulation of energy in the system, since the external force at each moment acts in time with the oscillation of the body.

Even a small periodic action (disturbing force) can cause large amplitudes of motion if the damping (energy losses) is insignificant.

The frequency of natural (free) oscillations of the system, consisting of a working body of a vibrating machine with a mass  $m$ , which is installed on an elastic suspension with stiffness  $c$ , can be determined from the dependence:

$$\omega_0 = \sqrt{\frac{c}{m}}. \quad (1)$$

At the same time, the productivity of vibrating machines increases with the increase in the frequency of oscillations of their working bodies [1, 4]. Therefore, for resonant vibrating machines, it is necessary to increase the frequency of natural (free) oscillations  $\omega_0$ . According to (1), with a constant mass of the working body of the vibrating machine  $m$ , this can be done by increasing the stiffness of its elastic suspension  $c$ .

Therefore, for reliable vibration isolation of the working body of the resonant vibrating machine from its frames, the elastic suspensions of vibrating machines must be soft enough, and to increase their productivity, their stiffness must be as high as possible.

Therefore, when choosing the stiffness of the elastic suspension of vibrating machines, it is necessary to take into account the design features of these machines and the features of the technological process that they implement in each specific case, guided by the above considerations.

**Analysis of recent research and publications.** The work [5] is devoted to the study of the influence of elastic-damping properties of rubber-metal elements on the dynamic behavior of vibrating machines. It is shown that the use of rubber-metal links significantly changes the resonant properties of the system, reducing dynamic loads and improving vibration isolation; the optimal stiffness and damping parameters are determined to ensure stable operation of the machine.

In [6] it is proven that one of the ways to vibration isolate such systems from the environment is to use dynamic methods to eliminate vibrations. The work analyzes the influence of different types of suspension and the ratio of body mass to the mass of the conveyor chute on the forces transmitted to the soil.

The work [7] concerns the analysis of vibrations in vibrating machines, taking into account flexible (elastic) drive elements. The research focuses on how the elasticity of these elements affects the dynamics of the system, as well as on the development of methods for modeling and controlling

vibration processes to improve their efficiency and reliability.

In [8], a comprehensive assessment of the latest developments in nonlinear isolators in the absence of active control is presented. Various methods for reducing the resonant frequency of the isolator are described. The base isolation uses friction elements, laminated rubber bearings, and a friction pendulum. In [9], new experimental and numerical data on the dynamic stiffness of rubber isolators are presented, which is essential for the selection of rubber elastic elements in suspensions.

In [10], the design and modeling of metal-rubber isolators as a modern alternative to elastomers for heavy vibration mounts. [11] presents experimental data on how changing the dynamic stiffness of a rubber insulator changes the natural frequency of the structure, which directly affects the design of elastic suspensions.

**The purpose of the research** is to develop recommendations for the calculation and design of elastic suspensions for vibrating technological machines operating in both far-resonant and resonant modes, as well as elastic elements with adjustable stiffness for vibrating machines with an adjustable electromechanical wire.

**Presentation of the main material.** Currently, the most common type of elastic suspensions in vibration machines are helical cylindrical compression springs. They perceive not only longitudinal-axial compressive forces, but also transverse bending loads that arise during circular or elliptical trajectories of movement of working bodies. Such springs operate in the mode of multiple cyclic and shock dynamic loads, which cause inertial oscillations and possible co-impacts of turns.

For reasons of reliability and durability, suspensions based on helical cylindrical springs must satisfy a certain strength condition [1, 4]:

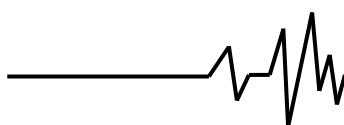
$$f_{cm} / A \geq 5, \quad (2)$$

where  $f_{st}$  – spring deformation in a static state;

$A$  – amplitude of oscillations of the upper end of the spring.

Condition (2) must also be met when choosing the design scheme and stiffness of the elastic suspension. Since the determination of the stiffness of the suspensions of U-shaped vibrating machines, consisting of two rows of springs located on both sides of the working bodies-containers, is quite simple and described in detail in special technical literature, it is advisable to focus further attention on the elastic suspension of toroidal vibrating machines.

In such machines, the working body - a toroidal container (see Fig. 1) - is fixed on the frame using elastic elements evenly spaced around the circumference with a radius  $R_0$ . Such a suspension scheme provides the possibility of arbitrary spatial oscillations of the container with a limited amplitude, while it has six degrees of freedom.



The coefficients of the total stiffness of the suspension of a toroidal container in the directions of the main axes and the corresponding displacements are determined by the following relations [1, 2, 4]:

$$C_z = \frac{Gd_n^4 n_n}{8\Delta_e^3 N_e}; \quad (3)$$

$$C_x = C_y = \frac{12EJ_n n_n}{\left[ N_e \left( t_0 - \frac{8Q\Delta_e^3}{n_n Gd_n^4} \right) \right]^3 \frac{2 + \sigma \cos \tau}{2 \sin \tau}}; \quad (4)$$

$$C_\varphi = \frac{12EJ_n R_0^2 n_n}{\left[ N_e \left( t_0 - \frac{8Q\Delta_e^3}{n_n Gd_n^4} \right) \right]^3 \frac{2 + \sigma \cos \tau}{2 \sin \tau}}; \quad (5)$$

$$C_\theta = C_\psi = \frac{12EJ_n n_n Z_k^2}{\left[ N_e \left( t_0 - \frac{8Q\Delta_e^3}{n_n Gd_n^4} \right) \right]^3 \frac{2 + \sigma \cos \tau}{2 \sin \tau}} + \frac{Gd_n^4 R_0}{8\Delta_e^3 N_e}; \quad (6)$$

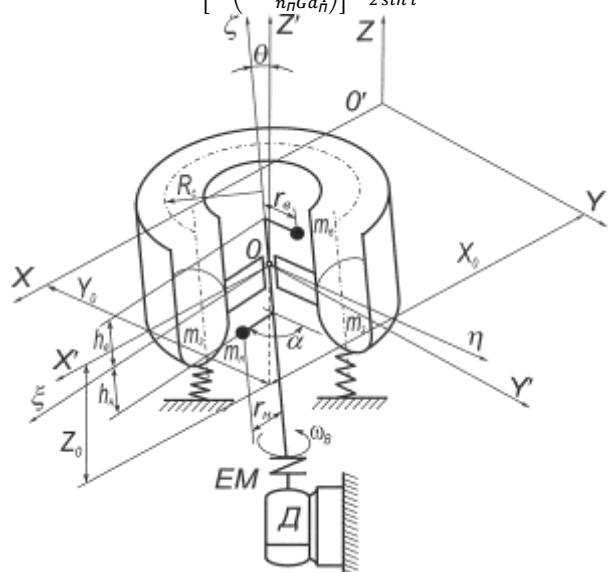


Fig. 1. Structural diagram of a vibrating machine with a toroidal container

where  $E$ ,  $G$ ,  $\sigma$  - modulus of elasticity of the first and second kind and the coefficient of transverse deformation of the spring material;

$d_n$ ,  $\Delta_e$ ,  $J_n$  - diameters of the wire and spring coil and axial moment of the wire cross-section;

$Q$  - weight of the vibrating parts of the vibration machine;

$N_e$  - number of spring coils;

$t_0$  - spring pitch in the free state;

$\tau$  - angle of elevation of the spring coil;

$n_n$  - number of springs;

$R_0$  - radius of the circle on which the springs are evenly placed.

Springs used in elastic suspensions of vibrating machines are made of steel spring wire or high-quality spring steel with a round, rectangular or trapezoidal profile [12]. When choosing a steel grade, it is necessary to take into account its hardenability,

suitability for surface hardening, availability of alloying elements and cost-effectiveness of production.

According to research results [12], the best performance characteristics for springs operating at temperatures up to 200 °C are provided by complex alloying of steels. A comparative analysis of strength and technological properties showed that 55KhGSF steel is optimal for suspensions of vibrating machines.

The durability and strength of springs are significantly affected by the quality of the surface. Any defects, scratches, dents, rust or decarburized layer sharply reduce endurance, especially under cyclic dynamic loads and coil impacts characteristic of the operation of vibrating machines.

Helix cylindrical springs are manufactured by cold or hot winding. Before the winding process, the wire is straightened, if necessary, on straightening and cutting equipment. After winding, the springs are subjected to mechanical processing of the ends, which must be perpendicular to the spring axis. To do this, in compression springs, the extreme turns are pressed and ground on surface grinding machines.

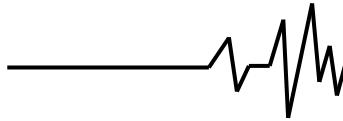
Springs made of carbon or alloy steels undergo heat treatment, which includes hardening and tempering. During hardening, intensive cooling is used in the temperature range of 650–400 °C, while the process can be slowed down later, especially in the range of 200–300 °C, where a martensitic structure is formed.

Tempering is used to reduce internal stresses that arise after quenching due to the heterogeneity of structural transformations in different zones of the material. High quality of heat treatment is ensured by uniform heating and cooling. For this purpose, chamber, muffle, gas or electric furnaces are used. Optimal steel characteristics are achieved only if the interaction of the metal surface with the environment during heating is minimized.

In the case of using 55KhGSF steel, during cold winding, the springs are subjected to only low tempering at a temperature of 200–300 °C. Such treatment contributes to the removal of residual stresses, deformation aging and redistribution of dislocations by the type of polygonization. As a result, the steel acquires increased elastic properties and a lower tendency to creep, although the brittleness increases somewhat. The optimal tempering temperature is determined by the purpose of the spring and the conditions of its operational load.

The optimal tempering temperature depends on the operating conditions of the springs: at small deformation amplitudes, it is advisable to use a lower temperature, since the requirements for plasticity in this case are less critical. On the other hand, at significant oscillation amplitudes, a higher tempering temperature should be used, which provides increased plasticity of the material.

To achieve a high elastic limit and resistance to small plastic deformations, isothermal quenching is used. Springs, preheated to the austenitic state, are



immersed in a medium (oil or heated salt) with a temperature slightly higher than the temperature of the onset of martensitic transformation. The holding continues until the austenite is completely transformed into acicular trostite, after which the products are cooled in air or in oil. As a result of such heat treatment, the material acquires increased viscosity, plasticity and reduced residual stresses, which is especially important for springs of suspensions of working bodies of vibrating machines operating under dynamic loads.

For steel grade 55KhGSFA, the following heat treatment regime is recommended [12]:

- isothermal quenching at a temperature of 870 °C;
- holding in a salt bath at 325–335 °C;
- tempering at 250 °C.

The holding time during isothermal quenching should be 20–30 min.

After heat treatment and mechanical grinding of the bearing surfaces, cylindrical helical springs are cleaned using hydrosandblasting equipment. After that, they are washed, passivated and dried. In order to increase corrosion resistance and extend the service life, the springs are covered with protective layers - metallic (tin plating, galvanizing), non-metallic (varnishes, paints, resins, lubricants) or chemical (phosphating, oxidation).

Practical experience shows that even springs made using the correct technology from high-quality wire can prematurely lose their elastic properties or break during operation - especially under dynamic loads typical of vibrating machines.

Studies [12] have shown that the decisive factor in endurance is the state of the surface layer, where maximum torsional and bending stresses occur during spring operation. This layer is significantly affected by external factors and the quality of surface treatment, which leads to the formation of micro-damages and irregularities that act as stress concentrators. Therefore, the use of surface hardening technologies is of key importance for increasing the endurance limit and increasing the service life of springs.

Hardening can be performed by mechanical, thermal or chemical-thermal action on the surface layer. High results are provided by mechanical hardening (surface slandering), which is carried out using shot-blasting, hydro-abrasive or vibration-hardening installations. The effect of such treatment is to improve the structure of the surface layer and the formation of residual compressive stresses, due to which the elastic limit, temporary resistance, hardness of the material increase and the tendency to stress relaxation decreases.

The main directions of further increasing the static and cyclic strength of springs include:

- creation of new grades of spring steels with improved physical and mechanical characteristics;

- development and implementation of modern technological methods of strengthening aimed at increasing the bearing capacity of springs;

- improvement of methods of design and calculation of spring endurance taking into account the real conditions of their operation.

To calculate the endurance of springs with an unlimited number of cycles, the following dependence has been proposed [12]:

$$\frac{\tau_v}{\tau_{-1}} + \frac{\tau_m}{\tau_B} = \frac{1}{n_r} \quad (7)$$

where  $n_r$  - is the safety margin;

$$\tau_v = \frac{\tau_{\max} - \tau_{\min}}{2} \quad \text{- amplitude of alternating voltage;}$$

$\tau_1$  - endurance limit at a symmetrical load cycle;  
 $\tau_B$  - temporary resistance;

$$\tau_m = \frac{\tau_{\max} + \tau_{\min}}{2} \quad \text{- average voltage per cycle;}$$

$\tau_{\max}$ ,  $\tau_{\min}$  - repeatedly repeated maximum and minimum voltages at the cross-sections of the spring turns.

However, the dependence (13) does not take into account the influence of inertial impacts between the turns of the spring, contact stresses, as well as bending stresses arising from the finite velocity of propagation of the deformation wave. For an accurate calculation of the durability of springs, it is necessary to use endurance curves constructed for a specific material taking into account the corresponding coefficient of asymmetry of the loading cycle.

In work [12], the dependence between the maximum stresses  $\tau_{\max 1}$  and  $\tau_{\max 2}$  in the loading cycles and the coefficients of asymmetry  $r_1$  and  $r_2$  for the same durability of the springs was established:

$$\tau_{\max 1} = \tau_{\max 2} \sqrt{\frac{(1-r_1)^2 + \eta(1-r_1^2)}{(1-r_2)^2 + \eta(1-r_2^2)}}, \quad (8)$$

where  $\tau_{\max 1}$  i  $\tau_{\max 2}$  - are the maximum stresses at which the springs failed after N loading cycles at asymmetry coefficients  $r_1$  and  $r_2$ , respectively;

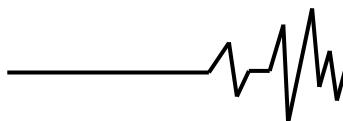
$\eta$  is the material sensitivity coefficient to asymmetry, which reflects the influence of the average stress level on durability.

The material sensitivity coefficient to asymmetry is not a constant value for a particular material, does not depend on the design parameters of the spring and is determined experimentally:

$$\eta = \frac{a^2(1-r_2)^2 - (1-r_1)^2}{(1-r_1^2) - a^2(1-r_2^2)}, \quad (9)$$

where  $a = \tau_{\max 2} / \tau_{\max 1}$ .

The service life of such springs A can be calculated using the following relationship:



$$A = \tau_{\max}^2 \cdot a_1 N^m, \quad (10)$$

where  $a_1 = [(1-r)^2 + \eta(1-r^2)]/4$ ;

$\tau_{\max}$  – maximum stress value in the cycle;

$N$  – number of cycles to failure;

$m$  – empirical exponent;

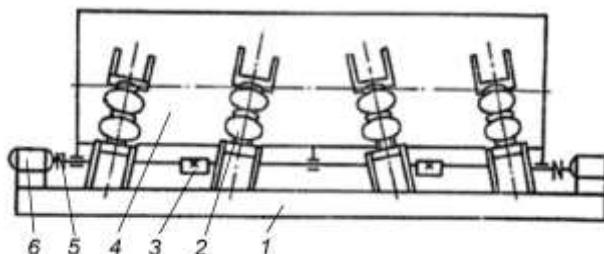
$r$  – cycle asymmetry coefficient.

When designing large-sized vibration machines with working containers with a volume of more than 150 dm<sup>3</sup> in elastic suspension systems, rubber-cord pneumatic cylinders (pneumatic springs) are widely used. They have a number of significant advantages:

- the ability to adjust the load-bearing capacity and stiffness in a wide range, which allows you to combine low stiffness with high bearing capacity and change the natural frequency of oscillations of the working containers of vibration machines;
- convenient adjustment of the suspension height by changing the pressure;
- practically silent operation;
- high durability and ease of maintenance and replacement.

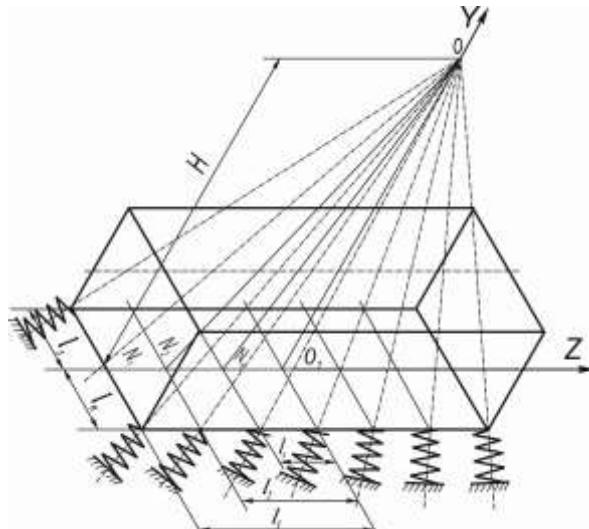
At the same time, air springs have a significant drawback - low transverse rigidity compared to longitudinal, which requires the use of additional elements to ensure the stability of the structure in the horizontal plane.

In the vibrating machine [13], air springs are installed symmetrically relative to the central vertical plane of the container at a certain angle to the vertical axis and to this plane (see Fig. 2, 3). It is worth noting that the location of the suspension elements at an angle to the transverse and longitudinal planes passing through the center of gravity of the container significantly increases the stability of the working body, especially during starting and stopping the machine.



1 – frame; 2 – pneumatic cylinder; 3 – unbalanced vibration exciter; 4 - container; 5 – elastic coupling; 6 – drive electric motor

**Fig. 2. Structural diagram of a vibrating machine with inclined pneumatic vibration supports**



**Fig. 3. Scheme for the location of pneumatic vibration mounts**

To ensure uniform load distribution between individual elastic elements, they are arranged in such a way that the lines drawn through the axes of symmetry of these elements intersect at one common point. This point should lie on the axis of symmetry of the container, passing through its center of gravity. In this case (see Fig. 4) we obtain:

$$H^2 = OO_1^2 = OM^2 - l_n^2. \quad (11)$$

The distance  $H$  can be calculated using the dependencies:

$$H = ON_1^2 - l_2^2 = ON_2^2 - l_2^2 = \dots = ON_s^2 - l_s^2. \quad (12)$$

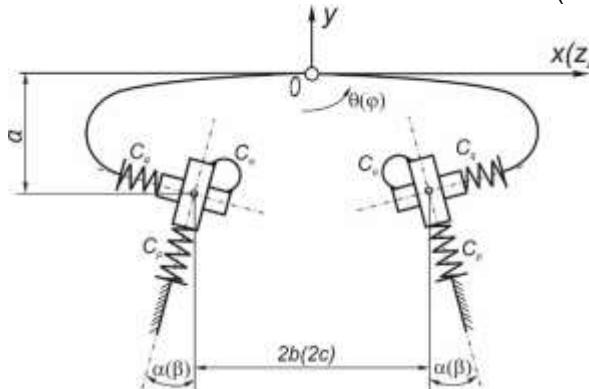
It should be noted that:

$$OM = l_1 \operatorname{tg} \beta_1 = l_2 \operatorname{tg} \beta_2 = \dots = l_s \operatorname{tg} \beta_s;$$

$$ON_1 = l_n \operatorname{tg} \alpha_1;$$

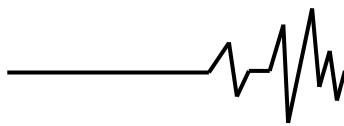
$$ON_2 = l_n \operatorname{tg} \alpha_2; \dots$$

$$ON_s = l_n \operatorname{tg} \alpha_s; \quad (13)$$



**Fig. 4. Calculation diagram of the pneumatic suspension of a vibrating machine**

From dependence (13) it is possible to determine the ratio between the angles of inclination of the axes of pneumatic elastic



elements in the longitudinal planes:

$$l_1 \operatorname{tg} \beta_1 = l_2 \operatorname{tg} \beta_2 = \dots = l_s \operatorname{tg} \beta_s. \quad (14)$$

Substituting expression (12) into (13), we have:

$$H = l_n^2 \operatorname{tg}^2 \alpha_1 - l_1^2 = l_n^2 \operatorname{tg}^2 \alpha_2 - l_2^2 = l_n^2 \operatorname{tg}^2 \alpha_s - l_s^2. \quad (15)$$

$$H = l_n^2 \operatorname{tg}^2 \beta_1 - l_1^2 = l_n^2 \operatorname{tg}^2 \beta_2 - l_2^2 = l_n^2 \operatorname{tg}^2 \beta_s - l_s^2. \quad (16)$$

By comparing expressions (15) and (16), we can find the dependences between the angles of inclination of elastic elements in the longitudinal  $\alpha_i$  and transverse  $\beta_i$  planes:

$$\frac{\operatorname{tg}^2 \beta_1 + 1}{\operatorname{tg}^2 \alpha_1 + 1} = \frac{l_n^2}{l_1^2}, i = 1 \dots 5. \quad (17)$$

Therefore, when designing vibration machines with an inclined arrangement of elastic suspension elements, to ensure uniform loading, it is necessary to select the angles of inclination of the axes of the elastic elements in the longitudinal plane in accordance with the relations (14). In this case, the angle of inclination of each element in the longitudinal plane is related to its inclination in the transverse plane by the dependence (17).

The use of this method of fastening elastic elements, especially in large vibration machines with a working container with a volume of more than 500 dm<sup>3</sup>, allows you to significantly increase their service life. According to this design scheme, machines with a container volume of 150, 250, 750 and 1000 dm<sup>3</sup> were created.

Let us consider the method for determining the stiffness of the elastic system in the case of installing elements at an angle. The design scheme of the vibration machine suspension is shown in Fig. 3. The elastic suspension is modeled as a hinge-lever mechanism with elastic links, with the upper kinematic pairs of pneumatic cylinders replaced by lower ones. The degree of mobility of the container in the cross-sectional plane is determined by the Chebyshev formula:

$$W = 3n - 2P_5 = 3 \cdot 5 - 2 \cdot 6 = 3.$$

This corresponds to the possibility of moving the working body along the axes OX, OY and its rotation relative to the axis OZ, which are taken as generalized coordinates.

For further determination of the generalized stiffnesses  $C_x$ ,  $C_y$ ,  $C_\theta$  and  $C_{x\theta}$ , the expression for the potential energy of the pneumatic cylinders in the unbalanced position of the container can be written. Let the suspension consist of eight pairs of elastic elements, each of which has a left and right element, located at an angle  $\alpha_i$  to the vertical longitudinal plane, then:

$$V = \sum_{i=1}^s V_i, \quad (18)$$

where  $V_i$  - is the potential energy stored in the  $i$ -th pair of elastic elements, which is equal to:

$$V_i = \frac{1}{2} \left\{ \begin{array}{l} C_p \left[ (\Delta P_i^{\text{II}})^2 + (\Delta P_i^{\text{I}})^2 \right] + \\ + C_q \left[ (\Delta q_i^{\text{II}})^2 + (\Delta q_i^{\text{I}})^2 \right] + \\ + C_\varphi \left[ (\Delta \theta_i^{\text{II}})^2 + (\Delta \theta_i^{\text{I}})^2 \right] \end{array} \right\}, \quad (19)$$

where  $\Delta P_i^{\text{II}}$ ,  $\Delta P_i^{\text{I}}$  - respectively, the deformation value of the left and right pneumatic cylinders of the  $i$ -th pair, which are equal to:

$$\begin{aligned} \Delta P_i^{\text{II}} &= (x + a\theta) \sin \alpha_i + (y - b\theta) \cos \alpha_i; \\ \Delta P_i^{\text{I}} &= -(x + a\theta) \sin \alpha_i + (y + b\theta) \cos \alpha_i; \\ \Delta q_i^{\text{II}} &= -(x + a\theta) \cos \alpha_i + (y - b\theta) \sin \alpha_i; \\ \Delta q_i^{\text{I}} &= (x + a\theta) \cos \alpha_i + (y - b\theta) \sin \alpha_i; \\ \Delta \theta_i^{\text{II}} &= \Delta \theta_i^{\text{I}} = \Delta \theta. \end{aligned}$$

Taking into account these expressions, dependence (19) will have the form:

$$V_i = \frac{1}{2} \left\{ \begin{array}{l} 2x^2 (C_p \sin^2 \alpha_i + C_q \cos^2 \alpha_i) + \\ + 2x^2 (C_p \cos^2 \alpha_i + C_q \sin^2 \alpha_i) + \\ + 2\theta^2 \left[ C_p (a \sin \alpha_i + b \cos \alpha_i)^2 + \right. \\ \left. + C_q (a \cos \alpha_i + b \sin \alpha_i)^2 + C_\varphi \right] + \\ + 2x\theta \sin 2\alpha \left[ C_p (a \operatorname{tg} \alpha_i - b) + C_q (a \operatorname{ctg} \alpha_i - b) \right] \end{array} \right\}. \quad (20)$$

The coefficients written in the squares and products of generalized coordinates in the curly brackets of expression (20) are generalized stiffness:

$$C_{x_i} = 2(C_p \sin^2 \alpha_i + C_q \cos^2 \alpha_i),$$

$$C_{y_i} = 2(C_p \cos^2 \alpha_i + C_q \sin^2 \alpha_i),$$

$$C_{\theta_i} = 2 \left[ \begin{array}{l} C_p (a \sin \alpha_i + b \cos \alpha_i)^2 + \\ + C_q (a \cos \alpha_i + b \sin \alpha_i)^2 + C_\varphi \end{array} \right];$$

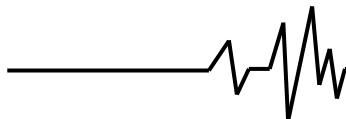
$$C_{x\theta_i} = 2 \sin 2\alpha \left[ C_p (a \operatorname{tg} \alpha_i - b) + C_q (a \operatorname{ctg} \alpha_i + b) \right]$$

The appearance of stiffness  $C_{x\theta_i}$  is associated with the non-coincidence of the center of gravity of the system with the center of its mass, therefore the horizontal movement of the container is associated with its rotation.

The values of the total generalized stiffnesses can be calculated using the expressions:

$$C_x = \sum_{i=1}^s C_{x_i}, \quad C_y = \sum_{i=1}^s C_{y_i},$$

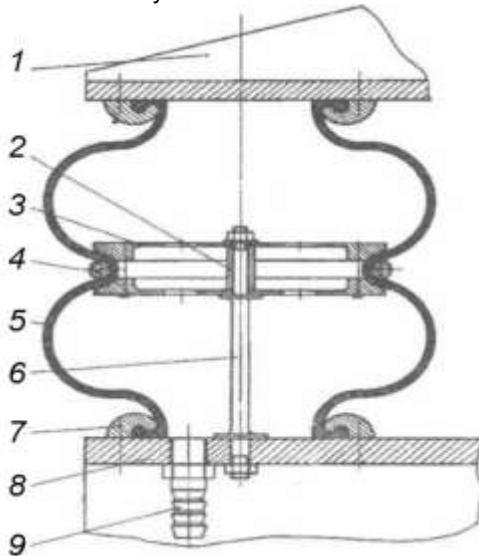
$$C_\theta = \sum_{i=1}^s C_{\theta_i}, \quad C_{x\theta} = \sum_{i=1}^s C_{x\theta_i}.$$



A special place among vibration machines is occupied by installations of the resonant type, which are distinguished by a number of technological, economic and operational advantages compared to other types. However, their significant disadvantage is the dependence of dynamic characteristics on the level of technological loading.

In this regard, the use of elastic suspensions with adjustable stiffness is of considerable interest, which makes it possible to ensure the stability of the dynamic mode of operation of the vibration machine and, if necessary, change it according to a predetermined law.

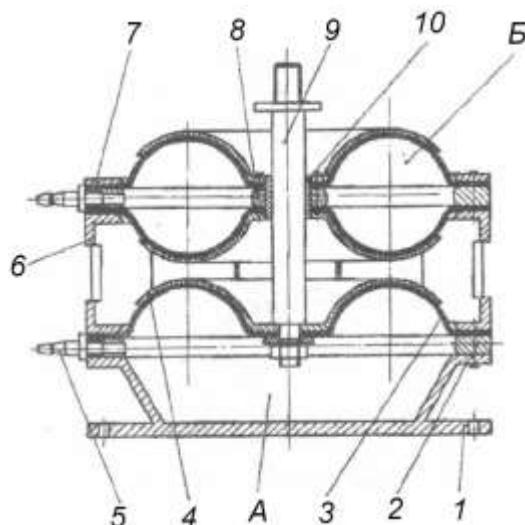
The design of a combined elastic suspension of vibration machines, which has increased transverse stiffness of double-ribbon pneumatic springs, is shown in Fig. 5 [13]. In the internal cavity of the pneumatic spring, mounted on the stiffness ring 4, there are elastic membranes 3, which are connected to the frame 8 by means of an elastic rod 6. In such elastic suspensions, the transverse stiffness is regulated by selecting the parameters of the elastic elements connecting the base with the body.



1 – container of the vibrator; 2 – spacer sleeve; 3 – elastic membrane; 4 – ring; 5 – pneumatic cylinder; 6 – elastic rod; 7 – flange; 8 – frame; 9 - fitting;

**Fig. 5. Combined elastic suspension with elastic membranes and elastic rod**

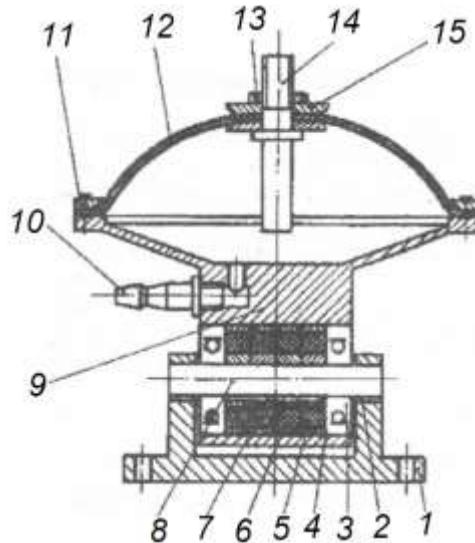
Fig. 6 shows the design of pneumatic elastic elements in which the stiffness can be separately adjusted in the longitudinal and transverse directions [13].



1 – housing; 2 – ring; 3- elastic diaphragm; 4 – spherical washer; 5 – fitting; 6 – intermediate housing; 7 – washer; 8 – sleeve; 9 – rod; 10 - bolt

**Fig. 6 Design of a pneumatic elastic element with separate stiffness adjustment**

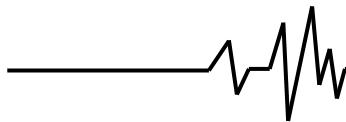
A promising option is also combined elastic suspensions that combine a diaphragm pneumatic actuator and a rubber-metal block hinge (Fig. 7) [13]. The suspension structure includes a frame 1, on which a housing 9 is mounted using an axis 8 and bearings 3. Rubber-metal block hinges 5 are fixed to the axis using a key 7, the outer rings 6 of which are connected to the housing 9. A diaphragm 12 is installed in the upper part of the housing, equipped with a pin 14 for attachment to the container of the vibrators.



1 – bracket; 2 – key; 3 – bearing; 4 – key; 5, 6 – metal bushing; 7 – rubber bushing; 8 – axis; 9 – housing; 10 – fitting; 11 – ring; 12 – diaphragm; 13 – nut; 14 – stud; 15 - washer

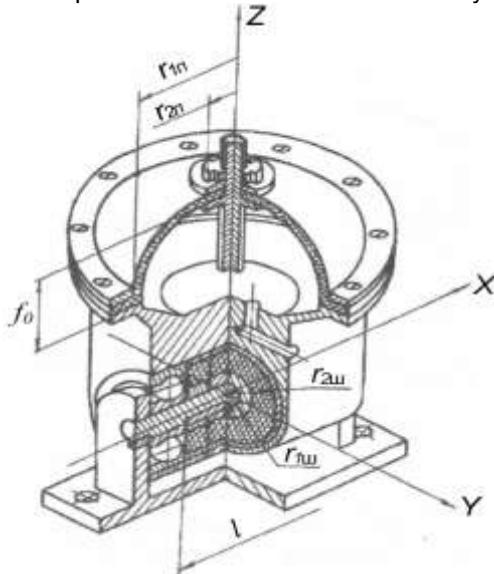
**Fig. 7. Design of a pneumatic elastic element with a diaphragm and a rubber-metal block hinge**

During the operation of the vibrating machine, the diaphragm mainly perceives vertical



vibrations of the container, while the block hinges - torsional. Due to the fact that only elastic torsional deformations occur in the rubber bushings 5, optimal operating conditions for the rubber-metal block hinges are provided. The torsional stiffness of the suspension can be adjusted by changing the number of block hinges, and the stiffness of the diaphragm - by changing the gas pressure in the suspension cavity.

Fig. 8 shows a calculation diagram of an elastic suspension with a combined elastic system.



**Fig. 8. Calculation scheme of an elastic suspension with a combined elastic system**

Let us consider the calculation method of an elastic suspension with a combined elastic system. The purpose of the calculation is to determine the total stiffness of the elastic elements. To do this, it is necessary to calculate six stiffness components - three translational along the axes OX, OY, OZ and three torsional relative to these same axes.

A rubber-metal block-hinged joint (RBH) can operate in the modes of coaxial torsion, axial displacement and perception of radial load. In this case, the RBH operates in coaxial torsion, and its stiffness is determined by the formula [13]:

$$C_K = \frac{4\pi G l r_{1III}^2 r_{2III}^2}{r_{2III}^2 - r_{1III}^2}. \quad (21)$$

The stiffness of the diaphragm in the axial direction can be determined from the dependence:

$$C = \frac{\pi P r_{1III}^2}{3 f_0} (1 + \rho_2 + \rho_2^2), \quad (22)$$

where  $P$  - air pressure;

$$\rho = \frac{r_{2III}^2}{r_{1III}^2}.$$

The other five components of the diaphragm stiffness are much smaller compared to the stiffness in the axial direction, so they can be taken equal to zero. The diaphragm is characterized by high load-bearing capacity and the possibility of adjusting the axial stiffness. The working vibrations of the container of the vibrator occur in the YOZ plane.

The stiffness of the elastic suspension in the vertical direction (along the OZ axis) corresponds to the stiffness of the diaphragm, while in the horizontal direction (along the OY axis) it is determined by the stiffness of the RBH during coaxial torsion.

Let us find the stiffness of the RBH, reduced to the point O, located at a distance  $h$  (Fig. 12) from its axis. The displacement of the point  $Y_p$  when the RBH is rotated by an angle  $\varphi$  is  $h\varphi$ . The force reduced to this point, acting in the direction of movement from the moment  $M$  in the RBH, is equal to  $F = M/h$ .

The reduced stiffness can be determined by the expression:

$$C_K = \frac{F}{Y_H}. \quad (23)$$

The magnitude of the angle of rotation of the RBH under the action of the moment  $M$  can be determined by the expression:

$$\varphi = \frac{M}{4\pi G l} \cdot \frac{r_{2III}^2 - r_{1III}^2}{r_{2III}^2 \cdot r_{1III}^2}. \quad (24)$$

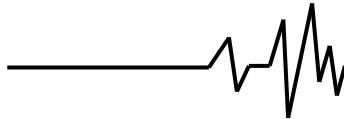
By substituting the obtained values into dependence (23), we can obtain an expression for calculating the stiffness reduced to point O:

$$C_K = \frac{4\pi G l r_{1III}^2 r_{2III}^2}{h^2 (r_{2III}^2 - r_{1III}^2)}. \quad (25)$$

In addition to translational movements along the OZ and OY axes in the ZOY plane, the elastic connection also provides the possibility of rotational movement in this plane. The stiffness of the elastic suspension during such movements is determined by the formula (22).

Usually, in vibration machines, the container is installed not on one, but on several elastic suspensions. Most often, two suspensions are placed in the plane of oscillations, while other elements of the elastic system prevent the container from rotating in this plane, allowing only movement along the OZ and OY axes.

When designing a vibration machine, the operating frequency of the container's oscillations is specified in the technical task. Having approximately determined the mass of the moving parts of the machine, the required stiffness of the elastic suspensions is calculated using these parameters.



For resonant vibration machines with an adjustable electromechanical oscillation drive:

$$C = \omega_p^2 m, \quad (26)$$

where  $\omega_p$  - circular frequency of oscillations of the moving masses of the vibrator;

$m$  - mass of the vibrating parts of the vibrator.

For vibrators with a non-resonant operating mode:

$$C = \left( \frac{\omega_p}{K} \right)^2 m, \quad (27)$$

where  $K$  - is the coefficient of vibration machine detuning from resonance ( $K = 3 \dots 5$ ).

Expressions (24) and (25) allow us to calculate the value of the total stiffness of all elastic suspension elements brought to the center of mass of the vibrating parts of the vibration machine.

By multiplying the right part of expression (21) by the number of suspensions  $n$  and equating it with the right part of expression (24), we can obtain an expression for the dependence of the geometric parameters of the diaphragm on the design parameters of the vibration machine, which is also valid for vibration machines operating in the resonant mode:

$$r_{1II}^2 (1 + \rho_2 + \rho_2^2) = \frac{3f_0 \omega_p^2 m}{\pi P n}. \quad (28)$$

The total horizontal stiffness RBH, brought to the center of mass of the vibrating parts of the vibrating machine, can be determined using expression (25):

$$C_K = \frac{4n\pi G l r_{1III}^2 r_{2III}^2}{h^2 (r_{2III}^2 - r_{1III}^2)}. \quad (29)$$

By comparing the right-hand sides of expressions (26) and (29), we can obtain the necessary ratios for calculating the geometric dimensions RBH:

$$\frac{r_{1III}^2 r_{2III}^2}{(r_{2III}^2 - r_{1III}^2)} = \frac{\omega_p^2 m h^2}{4n\pi G}. \quad (30)$$

For vibration machines operating in off-resonant modes, expressions (28) and (30) will take the form:

$$r_{1II}^2 (1 + \rho_2 + \rho_2^2) = \frac{3f_0 \omega_p^2 m}{\pi P K^2 n}. \quad (31)$$

$$\frac{r_{1III}^2 r_{2III}^2}{(r_{2III}^2 - r_{1III}^2)} = \frac{\omega_p^2 m h^2}{4n\pi G K^2}. \quad (32)$$

**Conclusions.** Recommendations have been developed for the calculation and design of elastic suspensions of vibrating technological machines operating in both far-resonant and resonant modes, as well as elastic elements with adjustable stiffness for vibrating machines with an adjustable electromechanical drive. Cylindrical helical compression springs are advisable to use for relatively small vibrating

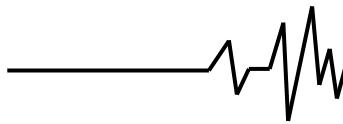
machines with a container volume of not more than 100 dm<sup>3</sup> and to install them without tilting. In large vibrating machines, it is desirable to use combined elastic suspensions or suspensions in the form of rubber-cord pneumatic cylinders, installing them at an angle to the longitudinal and transverse planes.

A special place among vibrating machines is occupied by installations of the resonant type, which are distinguished by a number of technological, economic and operational advantages compared to other types. However, their significant disadvantage is the dependence of dynamic characteristics on the level of technological loading.

In this regard, the use of elastic suspensions with adjustable stiffness is of considerable interest, which allows to ensure the stability of the dynamic mode of operation of the vibratory machine and, if necessary, to change it according to a predetermined law. A promising option is also combined elastic suspensions that combine a diaphragm pneumatic drive and a rubber-metal block hinge. The torsional stiffness of such a suspension can be adjusted by changing the number of block hinges, and the stiffness of the diaphragm - by changing the gas pressure in the suspension cavity.

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### ПРУЖНІ ПІДВІСКИ ВІБРАЦІЙНИХ МАШИН З РЕГУЛЬОВАНИМ ЕЛЕКТРОМЕХАНІЧНИМ ПРИВОДОМ

Високі вимоги до надійності, довговічності, потужності та продуктивності вібраційних машин безпосередньо визначаються якістю їхніх пружних елементів, зокрема їхньою витривалістю та стійкістю до дії інтенсивних циклічних навантажень. Для ефективної віброізоляції робочого органу резонансної

вібромашини від станини підвіски повинні мати достатню м'якість, водночас підвищення продуктивності потребує максимально можливої жорсткості цих елементів.

У роботі наведено рекомендації щодо розрахунку та проектування пружних підвісок вібраційних технологічних машин, що функціонують як у за-резонансному, так і в резонансному режимах, а також пружних елементів із регульованою жорсткістю для машин із регульованим електромеханічним приводом. Циліндричні гвинтові пружини стиску доцільно застосовувати у відносно малих вібраційних установках з об'ємом контейнера до 100 дм<sup>3</sup>, встановлюючи їх без нахилу. Для потужніших машин рекомендується використання комбінованих підвісок або гумо-кордових пневмобалонів, розміщених під кутом до повздовжньої та поперечної площин.

Особливе значення мають вібраційні машини резонансного типу, які вирізняються рядом технологічних, енергетичних і експлуатаційних переваг, але водночас характеризуються залежністю динамічних параметрів від ступеня завантаження.

З огляду на це, перспективним напрямом є використання підвісок із регульованою жорсткістю, що забезпечують стабільність динамічного режиму роботи вібромашини та дозволяють змінювати його відповідно до заданого алгоритму. Ефективним рішенням є також комбіновані пружні підвіски, які поєднують діафрагмовий пневмопривод із гумо-металевим блок-шарніром. Крутільну жорсткість таких систем можна змінювати регулюванням кількості блок-шарнірів, а жорсткість діафрагми — зміною тиску газу в порожнині підвіски.

**Ключові слова:** пружні підвіски, вібраційна машина, регульований електромеханічний привод, жорсткість, власна частота коливань, резонансний режим, пневмобалон.

### Відомості про авторів

**Yaroshenko Leonid** Candidate of Technical Sciences, Associate Professor of the department of electric power engineering, electrical engineering and electromechanics of Vinnytsia National Agrarian University, Service address: Vinnitsa, st. Sonyachna 3, VNAU 21008, e-mail: volvinlv@gmail.com

**Slobodianyk Anatoliy** Candidate of Technical Sciences (PhD), Associate Professor, Associate Professor of the Department of Mathematics, Physics and Computer Technologies, Vinnytsia National Agrarian University, Service address: Vinnitsa, st. Sonyachna 3, VNAU 21008, <https://orcid.org/0009-0006-2157-8188>

**Ярошенко Леонід Вікторович** кандидат технічних наук, доцент кафедри електроенергетики, електротехніки та електромеханіки Вінницького національного аграрного університету, Службова адреса: г. Вінниця, вул. Солнячна 3, ВНАУ 21008, e-mail: volvinlv@gmail.com

**Слободяник Анатолій Дмитрович** кандидат технічних наук, доцент, доцент кафедри математики, фізики та комп'ютерних технологій Вінницького національного аграрного університету, Службова адреса: г. Вінниця, вул. Солнячна 3, ВНАУ 21008, <https://orcid.org/0009-0006-2157-8188>

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