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IMPULSE DYNAMIC PROCESSES AND WAVE PHENOMENA IN THE CATERPILLAR MOVER OF THE TERRESTRIAL ROBOTIC COMPLEX

The results of investigations of dynamic wave processes in a caterpillar mover of a terrestrial robotic complex are presented. For this purpose, the theoretical model of the dynamics of caterpillar mover is substantiated. The model takes into account the occurrence of dynamic pulsed loads in the caterpillar mover drive. The loads are described by a random process through random amplitude periodic impulses. The dynamic caterpillar system is presented as a huge infinite tension thread grounded on an elastic and strain basis.

The statistical characteristics of random loads in the caterpillar mover drive are determined. Spectral density of loads is found as a superposition of a continuous, fading with the growth of component frequency and pulsed resonance areas described by Gauss curves.

It is proved that pulsed loads in the drive and perturbation from the roadway cause the appearance of wave phenomena in the caterpillar through longitudinal waves propagated along the caterpillar causing its transverse oscillations. The energy dissipation in the caterpillar smoothes out the shape of the wave front and reduces the amplitude of oscillations.

Recommendations for improving the dynamic characteristics of a caterpillar mover in particular by using oscillation inertial damping, have been developed.

Keywords: mobile robot, chassis, dynamics, pulsed loads, spectrum, caterpillar, longitudinal waves, mathematical model, periodic oscillations.

Formulation of the problem. Terrestrial robotic complexes are used in conducting works with hazardous objects and in military affairs. Research and development of terrestrial robotic systems is important for the national security of Ukraine.

Terrestrial robotic systems have a caterpillar chassis where a manipulator and necessary devices and tools are located. Terrestrial robotic complexes allow speed of displacement up to 20 ... 30 km / h. At the same time, the speed of the caterpillar mover reaches 20 m / s and higher, and there are significant dynamic loads in the dynamic system of the caterpillar mover. Determining the characteristics of these loads is an actual scientific and technical problem. Its solution is important for the development of high-performance terrestrial robotic systems.

The general problem is to create a reliable caterpillar chassis of the terrestrial robotic complex. The problem is related to important scientific and technical tasks of importance for the national security of Ukraine.

Analysis of recent research and publications. Recent studies and publications of Kucherov D.P., Huslyakov O.M., Strutyns'kyy V.B. describe a number of designs of terrestrial robotic complexes of special purpose [1-3]. The results of investigations of Rybak L., A Casas-Diaz, M.H. Korayem of the main components of complexes, manipulators in particular [4, 5, 6], are presented. Terrestrial robotic complexes have small dimensions. Therefore, the caterpillar chassis works in a mode that is different from the traditional one for caterpillar vehicles - Alexandrov E. E. [7].

In the literature Dushchenko V.V. and Hvorost A.G. [8, 9] show that we could observe results of studies of geometrical and power parameters of vehicles with caterpillar movers. Features of dynamic vibrational processes in caterpillar movers are determined by Volosnikov...
S.A. and Koynash V.A. by means of mathematical modeling, [10,11]. However, these studies relate to heavy weight caterpillar movers and do not take into account random loading parameters. Some publications of Hyun-Min Joe describe random processes in robotic complexes [12] but they do not relate to the dynamics of the mover.

It is concluded on the basis of the performed analysis of information sources that there are no results of research of pulsed dynamic processes in drives and supporting parts of caterpillar movers of terrestrial robotic systems in the literature.

Therefore, previously unsettled parts of the general problem include the study of pulsed dynamic processes in the drive and the supporting part of the caterpillar movers of the terrestrial robotic complex.

**The purpose.** The purpose of the research described in this article is to determine the patterns of impulse dynamic processes in the drive and the supporting part of the caterpillar movers of the terrestrial robotic complex.

The objectives of the research are to substantiate the theoretical model of the dynamics of a caterpillar mover, to determine the pulsed characteristics of random loads in the drive, and to estimate the wave phenomena in the caterpillar movers of the terrestrial robotic complex.

In the research process, the theoretical methods of research are mainly applied on the basis of the theory of random processes and the theory of oscillations of continuum systems with distributed parameters.

**Main results of the research.** The terrestrial robotic complex has a platform with a caterpillar mover which has a manipulator, a video surveillance system and special equipment (Fig. 1).

The caterpillars of the robotic complex are made in the form of ring bands moving by means of drive gears. The caterpillars are flexible enough to smoothly bending the drive and driven drum and caterpillar track rollers. Typically, the caterpillar mover of the vehicle has one or more rollers that press the bottom of the caterpillar to the roadway.

The upper part of the caterpillar is held in a stretched state by elastic forces in tensile state.

In some cases (as shown in Figure 1), additional supporting rollers are used. The number of supporting rollers depends on the design of the caterpillar. In separate devices 12 or more supporting rollers are used (Fig. 2).

When moving the platform in the upper and lower parts (branches) of the caterpillar significant efforts occur. It is assumed that all parts of the caterpillar are a stretched heavy thread with appropriate elastic tensile characteristics and in the absence of resistance to bending. The caterpillar moves as a result of the rotation of the drive drum, the protrusions of which interact with the protrusions of the caterpillar (Fig. 3).
When rotating the drum, periodic intermittent loads occur in a caterpillar. When interacting with a caterpillar with uneven road surface, transient (shock) and established dynamic loads occur. Together with the efforts in moving the caterpillar, they form non-stationary dynamic processes in the caterpillar. The theoretical model of the drive of the caterpillar mover and the scheme of interaction of the caterpillar with the road surface are constructed on the basis of qualitative analysis of dynamic processes in the caterpillar chassis. It is assumed that the tensile forces in the branches of the caterpillar FH(t) have a constant (static) component F0 and a dynamic pulsed component Fi(t) due to the intermittent contact of the drum protrusions with protrusions on the surface of the caterpillar (Fig. 4).

The periodic nature of the interaction of the drum protrusions with the caterpillar protrusions causes the occurrence of periodic pulsed loads in the caterpillar Fi(t), which are of a random nature. Pulsed dynamic loads in the caterpillar mover are approximated by periodic impulses of random amplitude. It is assumed that the periodic load from the drum to the caterpillar represents a random process in the form of rectangular pulses of uniform length, the amplitude of which varies as a random variable with the normal distribution law (see Figure 4 b). The period of the impulses $T$ is taken as a constant, and the impulse width is designated as $\gamma T$.

The lower part of the caterpillar is located on the surface of the road and interacts with it. The external surface protrusions of the caterpillar have considerable malleability. Therefore, their interaction with the surface of the roadway is accompanied with an elastic-dissipative contact. Taking into account that the support rollers and drive drums provide a uniform clamping of the caterpillar to the surface of the roadway, the lower part of the caterpillar can be considered a tensile thread (string) that is located on an elastic basis. In order to simplify the calculation scheme, it is assumed that the elastic-dissipative properties of the foundation are constant along the length of the caterpillar. Accordingly, a scheme of an infinite stretched heavy thread (strings) on an elastic-dissipative basis was adopted for the study of dynamic processes in the caterpillar (Fig. 5).
The interaction of the caterpillar protrusions with the roadway by its physical nature is of impulsive (shock) origin. There are isolated protrusions on the surface of the roadway which also cause shock loads on the caterpillar. Therefore, it is assumed that the cause of the dynamic fluctuations of the caterpillar is the pulsed loads that act at different points of the caterpillar. Shock loads in the front section of the caterpillar are the most significant dynamic action. Caterpillar drive is subjected to impulse loads. Therefore, the scheme of the occurrence of dynamic processes in the caterpillar, which causes the impulse (shock) load, is common one.

As a result of pulsed loads in a caterpillar stretched considerable effort, wave processes occur. To investigate wave processes in a caterpillar mover, its mathematical model is designed as a system with distributed parameters. Thus, the proposed theoretical model of caterpillar mover dynamics of a terrestrial robotic complex includes the determination of periodic pulsed loads in the caterpillar drive and the discovery of the dynamic processes characteristics in the caterpillar that occur under pulsed loads on the roadway and on the drive as well.

**Determination of statistical characteristics of random loads in the caterpillar mover drive.**

![Fig. 6](image)

**Fig. 6.** Components of the random process of changing the load of the caterpillar in the form of periodic impulses of equal width and random amplitude, (a) and random impulses whose mathematical expectation is zero (b).

We define the spectral characteristic of the process as periodic rectangular impulses of constant amplitude. The process as periodic rectangular impulses is decomposed into an infinite Fourier series in the complex form [13]:

$$F_j(t) = \sum_{k=0}^{\infty} c_k \cdot e^{\frac{j2\pi k t}{T}}, \quad j = \sqrt{-1},$$

where $c_k$ - complex coefficients of a row.

Modules of complex coefficients are equal:

$$|c_k| = \frac{F}{k\pi} \sin(k\pi \gamma).$$

Let's consider separately the harmonic which enters into an infinite row:

$$F_k(t) = c_k e^{\frac{j2\pi k t}{T}}.$$

The impulse amplitudes of the load $F_i$ (see Fig. 4b) as random variables are characterized by mean and mid-quadratic values. The mean value of impulse amplitude will be determined as the arithmetic mean height:

$$\bar{F} = \frac{1}{N} \sum_{i=1}^{N} F_i,$$

where $N$ - a sufficiently large number of load impulses.

Impulse amplitude mid-quadratic value:

$$F = \sqrt{\frac{1}{N} \sum_{i=1}^{N} F_i^2}.$$

Let's expound a general random process (see Fig. 4b), which has the form of random rectangular periodic impulses of loading on two separate components:

$$F(t) = F_1(t) + F_2(t).$$

We choose the first component in the form of constant in the height of rectangular impulses, the amplitude of which is equal to the mean value of the output process (Fig. 6 a). The second component of formula (2) is defined as a centered random process in the form of alternating impulses of random amplitude (Fig. 6b).
delta functions decreases with increasing frequency according to the law:

\[ S_{10}(\omega) = \frac{F^2}{2\pi k^2} \sin^2(k\pi \gamma) \]

Considering the relationships:

\[ \omega = \omega_0 k, \quad \omega_0 = 2\pi / T, \quad \omega_k =\omega_0 k, \]

we define:

\[ k = \frac{\omega}{\omega_0} = \frac{T}{2\pi}. \]

Accordingly, changes in the intensity of delta functions:

\[ S_{10}(\omega) = \frac{2\pi F^2}{\omega_0^2 T^2} \sin^2(\frac{\omega T\gamma}{2}). \]

And we approximate the delta function of continuous dependence as the Gaussian curve

\[ \delta_a(\omega - \omega_k) = ae^{-\frac{(\omega - \omega_k)^2}{2\sigma_a^2}}, \]

where \( a \) – intensity of shock impulse; \( \sigma_a \) – parameter that determines the width of the impulse.

We define an integral from this dependence:

\[ \int_{\omega - \omega_k} \delta_a(\omega - \omega_k, \omega) \, dw = a\sigma_a \sqrt{2\pi}. \]

The spectral density of the total impulse load in the caterpillar mover is established. On the basis of the superposition principle the statistical characteristic of impulsive loads is found as the sum of the linear and continuous spectra (Fig. 7b).

The obtained integral value corresponds to the integral norm of the delta function:

\[ \int_{-\infty}^{+\infty} \delta(\omega - \omega_k) \, dw = 1. \]

Accordingly, we determine the intensity:

\[ a = \frac{1}{\sqrt{2\pi \sigma_a^2}}. \]

Ultimately, the approximation of the delta function by a continuous dependence will take the form:

\[ \delta_a(\omega - \omega_k) = \frac{1}{\sqrt{2\pi \sigma_a^2}} \cdot e^{-\frac{(\omega - \omega_k)^2}{2\sigma_a^2}}. \]

In this formula, the value of the parameter \( \sigma_a \) must be accepted small enough (\( \sigma_a \rightarrow 0 \)).

Taking into account the dependence (4), the spectral density (3) of the random process of loading in the drive of the catapillar mover corresponding to the component as permanent rectangular impulses will be determined by the following row:

\[ \delta_f(\omega) = \sum_{k=0}^{k=+\infty} \frac{2\pi F^2}{T^2 \sigma_a^2} e^{-\frac{(\omega - \omega_k)^2}{2\sigma_a^2}} \sin^2(\frac{\omega T\gamma}{2}). \]

According to formula (5), the value of the spectral density for the specific quantities \( F \) and \( \sigma_a \) (Fig. 7a) is calculated.

**Fig. 7. Statistical characteristics (spectral density) of impulse loads in the caterpillar mover drive:**

- a - spectral load density as periodic impulses of constant amplitude;
- b - spectral function corresponding to the sum of dependencies (5) i (6).

Spectral density has the shape of periodic frequency impulses whose intensity drops with increasing frequency.

We consider the second component of the \( F^2(t) \) impulse loads in the caterpillar drive (see Figure 6 b). The spectral density of the random \( F^2(t) \) component of the centered process in the shape of impulses of random amplitude according to [13] is determined by the dependence:

\[ S_2(\omega) = \frac{4\sigma^2}{T_0^2} \sin^2\left(\frac{\omega T}{2}\right) \cdot d\omega. \]

The spectral density of the total impulse load in the caterpillar mover is established. On the
We consider the transverse oscillations of the caterpillar as an infinite massive thread having a linear mass \( m \), stretched by the force \( F_R \) and placed on an elastic basis with a deformation coefficient \( c_R \) and a dissipative coefficient \( b_R \). We assume that at the time \( t = 0 \), at the point \( A_i \), the impulse load is applied to the caterpillar. Physically, this load corresponds to the appearance under the caterpillar of a single protrusion or cavity. A similar load occurs in the caterpillar when impulse changes in the drive or under the impact load of the roller.

We describe a pulsed load with an infinitely short impulse in the shape of a delta function:

\[ P = P_0 \cdot \delta(t) \]

The multiplier \( P_0 \) at the delta function determines the energy properties of the momentum.

The transverse displacements of the sections of the caterpillar as a massive stretched thread \( U = U(x, t) \) depend on the longitudinal coordinates \( x \) and vary in time \( t \). To determine transverse displacements of an infinite massive yarn, which is located on an elastic basis with a coefficient of rigidity of \( c_R \), we have a differential equation in partial derivatives in the shape [14]:

\[
m \cdot \frac{\partial^2 U}{\partial t^2} - F_R \cdot \frac{\partial^2 U}{\partial x^2} + c_R \cdot U = 0.
\] (7)

Equation (7) is the first approximation which does not take into account the dissipative properties of the caterpillar. The solution of equation (7) is expressed in terms of the Bessel function of the first kind of zero order in dependence:

\[
U(x,t) = \sum_{n=0}^{\infty} \frac{P_0 \cdot F_R}{2 \sqrt{F_R \cdot m}} \left( \frac{c_R}{F_R} \right)^{1/2} \sqrt{\frac{m}{F_R}} \left( \frac{F_R}{m} \right)^{1/2} \cdot x^{1/2} \cdot J_0 \left( \sqrt{\frac{m}{F_R}} \cdot \left( \frac{F_R}{m} \right)^{1/2} \cdot x \right).
\] (8)

Caterpillar intersection displacement that corresponds to this dependence determines a non-stationary wave-like process, which depends on two arguments of the distance of the caterpillar intersection from the point of impact of the impulse loading \( x \) and on time \( t \).

Under the action of an impact impulse in a caterpillar there is an impulse disturbance (wave front) which propagates through a caterpillar in the direction from the point of impact impulse action. After a certain time \( t \) following the impulse action, the caterpillar acquires a characteristic wavy shape ("running" wave) (Fig. 8)

![Fig. 8. The shape of the caterpillar is determined at different intervals of time \( \Delta t \) after the impact impulse action.](image)

The amplitude of the impulse \( F_1, F_2, F_3 \) decreases slightly over time. Insignificant impulse attenuation is explained by the dissipative properties of the caterpillar not taken into account in equation (7).

Displacement of different sections of the caterpillar, depending on the time, has the shape of damped wave processes (Fig. 9)

![Fig. 9. Dependence of transverse displacement of the caterpillar on time for different intersections.](image)
The abovementioned dependencies make it possible to estimate the velocity of the front wave displacement and changing its intensity. The laws of transverse dynamic displacements of separate sections of a caterpillar, their frequency composition, amplitude parameters and patterns of attenuation under action on a caterpillar single isolated impulse are also determined.

Various dynamic loads are acted on the caterpillar. As for the drive there are periodic pulsed loads of random amplitude. As for the roadway impulse loads act depending the speed of the chassis, the type of the profile of the roadway and the nature of the interaction of the protrusions on the surface of the caterpillar with the road. These pulsed loads are quasiperiodic. On the basis of it, it is assumed that random pulsed loads on the caterpillar operate at a certain interval in time, which is constant for the drive and variables for the roadblock. Thus, a package of impulses acts on the caterpillar. We assume that the impulses act at one point in the initial intersection of the caterpillar. The time of a single impulse - $t_k$, the impulse amplitude $P_{a_k}$ is a random variable with a normal distribution law.

The amplitude of the impulses is set by a random number generator with a normal distribution law:

$$P_{a_k} = \text{rnorm}(50,100,20).$$

On the basis of the superposition principle, the transverse vibrations of the caterpillar in the sections located at a distance $x$ from the origin of the coordinates are determined by the sum of the components, each of which is a solution of the equation (7). While caterpillar intersection displacement:

$$U(x,t) = \sum_{k=0}^{N} P_{a_k} \left[ \frac{C_R}{\sqrt{F_R}} \cdot \sqrt{\frac{F_R}{m} (t-t_k)^2 - x^2} \right]$$

while

$$\sqrt{\frac{F_R}{m} (t-t_k)} > x,$$

$$\sqrt{\frac{F_R}{m} (t-t_k)} < x,$$

where $N$ – total number of impulse loading.

According to dependence (9) transversal vibrations of the intersection of the caterpillar under the influence of periodic impulse loads in the actuator have been calculated (Fig. 10).

Fig. 10. Calculation of random displacements of intersections of a caterpillar under the influence of impulse loads in the drive.

It follows from the results of calculations that the presence of periodic pulsed loads in the drive causes the transverse resonance oscillations in the caterpillar.

The calculation is based on equation (7) without taking into account energy losses. Therefore, the implementation of random transverse oscillations has the form of pulses with sharply variable front (Fig. 11).

Fig. 11. Calculating dependences of transverse oscillations of a caterpillar intersection under the influence of random impulse loads: curve 1 - calculation by the formula (9); curve (2) by the formula (10); curve (3) by the formula (11).
The calculated shape of the impulses (curve 1) in certain periods in the form is close to the pulse characteristic of the dynamic system, which is described by the differential equation of the first order (see curve 2, Fig. 11).

This impulse characteristic is described by the dependence:
\[ w_k = B_k e^{-(t-t_k)/\tau_k} \cdot I(t-t_k), \quad (10) \]
where \( B_k, \tau_k \) - constants are chosen from the condition of the best approximation of dependencies (9) 1 (10) e within one period; \( t_k \) - the time of the corresponding impulse loading.

In formula (10), energy losses in the system are not taken into account. To account energy losses the impulse response (10) is presented in the shape of an impulse characteristic of the system described by the differential equation of the 2-nd order and could be presented as:
\[ w_D = \frac{Q_{ak}}{(\tau_{k3} - \tau_{k4})} \left[ e^{-(t-t_k)/\tau_{k3}} - e^{-(t-t_k)/\tau_{k4}} \right] I(t-t_k), \quad (11) \]
where constants \( Q_{ak}, \tau_{k3}, \tau_{k4} \) - are selected from the condition of the correspondence of dependencies (9), (11).

Calculation by formula (11) defines a smooth process of transverse oscillations of a caterpillar that has no rupture points (see Curve 3 in Figure 11).

Therefore, in order to calculate the transverse displacement of the caterpillar, the following dependence is recommended:
\[ U_D = \sum_{k=0}^{N} \frac{Q_{ak}}{\tau_{k3} - \tau_{k4}} \left[ \exp\left( -\frac{t-t_k}{\tau_{k3}} \right) - \exp\left( -\frac{t-t_k}{\tau_{k4}} \right) \right] \]
where \( Q_{ak} \) - random variable with normal distribution law.

Calculated transverse oscillations of the caterpillar are significant. The presence of oscillations limits the possibility of increasing the speed of the caterpillar, and, accordingly, the speed of the chassis.

Based on the research conducted, recommendations for improving the dynamic characteristics of the caterpillar mover have been developed. They include suggestions for improving the design of the caterpillar and the drive to move it. In order to improve the dynamic properties of the caterpillar mover significantly, it is proposed to use special inertial damping oscillations.

**Conclusions**
1. It has been found that periodic loads in the caterpillar mover drive and the interaction of the pruritions on the surface of the caterpillar with the roadway cause the pulsed perturbations of a wide frequency range in the caterpillar, the spectrum of which includes continuous, frequency-decaying, component and resonance regions similar to isolated impulses (delta function).
2. The effect of pulsed loads causes the occurrence of specific dynamic processes in the caterpillar that are wave-like and are accompanied by the propagation of longitudinal waves in a caterpillar whose intensity decreases over time. Wave phenomena substantially increase the load in the caterpillar.
3. Superposition of impulse dynamic loads in the caterpillar drive and the effect of the roadway form a stable quasiperiodic oscillation process in the caterpillar, which manifests itself in the occurrence of large amplitude transverse oscillations of the caterpillar intersections. To reduce the intensity of oscillation, it is recommended to use special dampers.

**References**

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