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STUDY OF GRAIN FLOW IN A CYLINDRICAL VIBRATING SIEVE

The paper proposes a modified hydrodynamic model of steady grain flow of a heterogeneous mixture on the surface of a vertical cylindrical vibrating sieve. The main hypothesis of the model is the dependence of the mixture's porosity in the moving annular layer on the particle velocity. A linear dependence is assumed, where an increase in velocity leads to an increase in the mixture's porosity. To determine the grain flow velocity, the problem is reduced to solving a non-homogeneous Bessel-type differential equation.

To simplify the solution, the "freezing" method of the variable coefficient is applied, which is permissible due to the relatively small thickness of the moving grain mixture layer compared to the sieve radius. As a result of this approximation, the velocity dependence on the radial coordinate is expressed using elementary functions. A compact formula for calculating the maximum grain flow velocity and the average velocity is derived by integrating the respective expression in elementary functions.

Additionally, an approximate formula for calculating the sieve's throughput based on the mass of the overflow fraction is proposed. Simpson's integration method is used to avoid complex calculations of special functions for large arguments via asymptotic formulas. The study shows that the sieve's throughput is significantly affected by the porosity of the grain mixture.

To assess the accuracy of the approximate formulas, numerical integration of the original Bessel-type differential equation was performed on a computer. Comparative analysis of the results confirmed that the proposed simplifications introduce minimal errors and ensure the adequacy of the theoretical results. Thus, by transitioning to the simplified differential equation, approximate formulas for calculating the key characteristics of grain flow in a vertical cylindrical vibrating sieve were developed and tested, taking into account the porosity variation of the mixture depending on the particle velocity.

The generalization of theoretical results obtained using hydrodynamic models of grain mixture flow in a pseudo-fluidized state under vibrations only slightly complicated the model but resulted in compact and practically convenient formulas for use.

Key words: *vertical cylindrical vibrating sieve, porosity dependence on velocity, "freezing" coefficient method, Bessel differential equation, grain flow velocity, sieve throughput.*

Introduction. The motion of the grain mixture on the vibrating sieve is described by various theories, starting from the simulation of motion in the form of a material point and ending with the model of a continuous medium. One of them is a hydrodynamic model that describes the motion of a fluidized medium. When the particles forming the mixture move, the porosity of the layer changes. This change affects the grain flow rate and, consequently, the performance of the vibrating sieves.

Analysis of recent research and publications. In the theory of motion of vibroseparated mixtures on the surfaces of sieves, a hydrodynamic model of a fluidized mixture, initiated in [1, 2, 3, 4, 5], has become widespread. These works were about the steady motion of a homogeneous annular layer on the inner surface of a vertical cylindrical vibrating screen that rotates around its axis. A generalization of this theory in the case of variable vibration viscosity of the mixture along the thickness of the layer was carried out in [2, 4, 6]. In [2, 7, 8, 9], the generalization of hydrodynamic models was made taking into account the change in porosity in the thickness of the moving layer due to changes in pressure. To do this, we compiled a separate differential equation describing the dependence of porosity on the spatial coordinate and solved it. Then the obtained solution was taken into account in the equation of motion of the mixture. But, as practice shows, the porosity of the mixture also depends on the speed of its movement. Based on this, an attempt is made to take into account this dependence, which is a kind of feedback, because the speed depends on the porosity, and the porosity - on the speed. The simplest variant of dependence is accepted in work, namely linear communication when porosity is big where the speed of movement is bigger. Previously, such a motion model was used when separating the mixture on a flat vibrating sieve [12, 13]. Here we consider the cylindrical shape of the sieve.

The purpose of article is the introduction and testing of approximate formulas for calculating the rate of steady motion of grain-mixed changes in porosity in the thickness of the layer during its movement on a vertically centrifugal cylindrical vibrating sieve.

Presentation of the main material. As in publications [1, 2], the original differential equation is:

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}=-\frac{\rho g}{\mu}. \qquad (1)$$

Here $u=u(r)$ – vertical, averaged over the period of oscillations, the speed of the annular layer; r – radial coordinate; ρ – specific weight of the grain mixture g – free fall acceleration; μ – dynamic coefficient of vibration viscosity, which depends on the parameters of the oscillations of the sieve and the characteristics of the grain mixture [4] or the concentration of particles [4, 7, 11, 14]. Boundary conditions to equation (1) are:

$$u(R)=0\,;\ \left.\frac{du}{dr}\right|_{r=R_0}=0, \qquad (2)$$

where R_0, R – respectively, the inner radius of the annular layer of the mixture and the radius of the sieve (Fig. 1).

In addition to (2), other variants of boundary conditions are possible [2], in the presence of additional mixture segregators on the surface of the sieve: ribs, reefs, etc. Summarizing the known theories, we accept:

$$\rho=\rho_*(1-\lambda\cdot u), \qquad (3)$$

and: $\lambda>0$; $\lambda\cdot u(R_0)<1$; ρ_* – specific weight of the grain mixture at rest.

Given (1), (3), we obtain a generalized differential equation of the Bessel type [15]:

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\gamma^2u=-\frac{\gamma^2}{\lambda}, \qquad (4)$$

where $\gamma=\sqrt{\rho_* g\lambda/\mu}$.

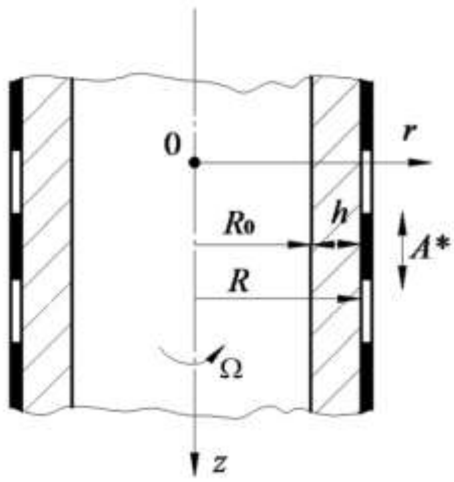

Fig. 1. Calculation scheme

The solution of equation (4) is expressed in cylindrical functions [16]. But in order not to calculate the values of special functions of a large argument by asymptotic formulas, we further construct approximate calculation formulas. To do this, instead of (4) we will solve the differential equation:

$$\frac{d^2u}{dr^2}+\frac{1}{r_*}\frac{du}{dr}-\gamma^2u=-\frac{\gamma^2}{\lambda}, \qquad (5)$$

in which $r_*=0,5(R_0+R)$.

Replacement (4) by equation (5) does not give large errors due to the fact that in the practice of separation, the thickness of the moving layer of the grain mixture is much smaller than the radius of the sieve. The effectiveness of this simplification is confirmed in [2, 4].

The general solution of differential equation (5) has the form:

$$u(r)=\frac{1}{\lambda}+C_1\exp(k_1r)+C_2\exp(k_2r), \qquad (6)$$

where $k_{1,2}=-\frac{1}{2r_*}\pm\sqrt{\gamma^2+\left(\frac{1}{2r_*}\right)^2}$; C_1, C_2 – arbitrary steels.

Substitution (6) in (2) gives a system of equations:

$$C_1k_1\exp(k_1R_0)+C_2k_2\exp(k_2R_0)=0,$$

$$C_1\exp(k_1R)+C_2\exp(k_2R)=-\frac{1}{\lambda}.$$

Solving this system, we obtain:

$$C_1=\frac{1}{\lambda}\frac{k_2\exp\left(k_2R_0\right)}{\Delta\left(R_0,R\right)};$$

$$C_2=-\frac{1}{\lambda}\frac{k_1\exp\left(k_1R_0\right)}{\Delta\left(R_0,R\right)}; \quad (7)$$

$$\Delta\left(R_0,R\right)=k_1\exp\left(k_1R_0\right)\exp\left(k_2R\right)-k_2\exp\left(k_1R\right)\exp\left(k_2R_0\right).$$

Given (6) and (7), we obtain the formula for grain flow rate:

$$u\left(r\right)=\frac{1}{\lambda}\left[1-\frac{k_1\exp\left(k_1R_0-k_2r\right)-k_2\exp\left(k_2R_0-k_1r\right)}{k_1\exp\left(k_1R_0-k_2R\right)-k_2\exp\left(k_2R_0-k_1R\right)}\right]. \quad (8)$$

The greatest value of speed is when $r=R_0$. Its calculation is reduced to the formula:

$$u\left(R_0\right)=\frac{1}{\lambda}\left[1-\frac{\left(k_1-k_2\right)\cdot\exp\left(k_1R_0+k_2R\right)}{\Delta\left(R_0,R\right)}\right].$$

Here, as well as in formula (8), there is uncertainty of type $\left|0/0\right|$ when $\lambda\to0$. Using the boundary transition in (8), we obtain for a homogeneous mixture ($\lambda=0$):

$$u\left(r\right)=\frac{\rho g r_*^2}{\mu}\left[\exp\left(\frac{R_0-R}{r_*}\right)-\exp\left(\frac{R_0-r}{r_*}\right)+\frac{R-r}{r_*}\right]. (9)$$

Earlier, this formula was derived in [2]. If you specify in it $r=R_0$, you get the maximum value of speed $u\left(R_0\right)$.

In the practice of separation to assess the performance of the vibrating screen using the values of the average speed [2, 9, 10, 11, 26]:

$$u_{av}=\frac{2}{R^2-R_0^2}\int_{R_0}^{R}r\cdot u\left(r\right)dr. \quad (10)$$

Given that [16]:

$$\int_{R_0}^{R}r\cdot\exp\left(kr\right)dr=\frac{1}{k^2}\left[\exp\left(kR\right)\cdot\left(kR-1\right)-\exp\left(kR_0\right)\cdot\left(kR_0-1\right)\right],$$

after substitution (8) in (10), we obtain

$$u_{av}=\frac{1}{\lambda}\left\{1+\frac{2\exp\left[\left(k_1+k_2\right)R_0\right]}{\left(R^2-R_0^2\right)\left(k_1k_2\right)^2\Delta\left(R_0,R\right)}\left[k_2^3f_1\left(R_0,R\right)-k_1^3f_2\left(R_0,R\right)\right]\right\}. \quad (11)$$

Here

$$f_1\left(R_0,R\right)=\left(k_1R-1\right)\exp\left[k_1\left(R-R_0\right)\right]+1-k_1R_0;$$

$$f_2\left(R_0,R\right)=\left(k_2R-1\right)\exp\left[k_2\left(R-R_0\right)\right]+1-k_2R_0.$$

When $\lambda\to0$ in (11) it is necessary to reveal the uncertainty of the species $\left|0/0\right|$. After performing this operation, we obtain:

$$u_{av}=\frac{\rho g r_*^2}{\mu}\left\{\begin{aligned}&\left[1+\frac{2r_*\left(R+r_*\right)}{R^2-R_0^2}\right]\exp\frac{R_0-R}{r_*}-2\frac{r_*\left(R_0+r_*\right)}{R^2-R_0^2}+\\&+\frac{R}{r_*}-\frac{2\left(R^2+RR_0+R_0^2\right)}{3r_*\left(R+R_0\right)}\end{aligned}\right\}$$

This is an approximate formula for the average velocity of a homogeneous mixture ($\lambda=0$).

When determining the mass productivity of the Q vibrating sieve by the stair fraction, it is necessary to calculate the integral:

$$Q = 2\pi\rho_* \int_{R_0}^{R} \left[1 - \lambda \cdot u(r)\right] u(r) r dr . \quad (12)$$

For dependence (8), it is expressed in elementary functions, but the answer is cumbersome and not convenient in practice. Therefore, having received six precision, the integral (12) can be calculated approximately by Simpson's formula [15]:

$$Q \approx \frac{\pi\rho_*(R - R_0)}{3} \times$$
$$\times \left\{\left[1 - \lambda \cdot u(R_0)\right] u(R_0) R_0 + 4\left[1 - \lambda \cdot u(r_*)\right] u(r_*) r_*\right\}. \quad (13)$$

The error of this approximation was due to the small thickness of the moving layer given by the difference $R - R_0$.

Results and Discussion. For calculations we accept $\rho_* = 750$ kg/m³; $R = 0{,}3075$ m; $R_0 = 0{,}2975$ m; $\mu = 0{,}6$ Pa·s and different λ. Obtained in two ways the values of velocity in the thickness of the annular layer at $\lambda = 0{,}1$ s/m are written in table. 1

Table 1

The value of speed at $\lambda = 0{,}1$ s/m

$10u(r)$, m	Form. (8)	Numbers. integral	$10u(r)$, m	Form. (8)	Numbers. integral.
	Value $10u(r)$, m/s			Value $10u(r)$, m/s	
2,975	5,7727	5,7727	3,025	4,3326	4,3327
2,985	5,7150	5,7150	3,035	3,6989	3,6990
2,995	5,5420	5,5420	3,045	2,9494	2,9495
3,005	5,2539	5,2540	3,055	2,0836	2,0837
3,015	4,8509	4,8509	3,065	1,1007	1,1008

As can be seen from table. 1, the numerical computer integration of the differential equation (4) confirmed the small errors of its replacement by the differential equation (5), which has simple solutions.

This conclusion confirms the comparative analysis and numerical results in table. 2, which are obtained at $\lambda = 0{,}9$ s/m.

Table 2

The value of speed at $\lambda = 0{,}9$ s/m

$10u(r)$, m	Form. (8)	Numbers. integral	$10u(r)$, m	Form (8)	Numbers integral
	Value $10u(r)$, m/s			Value $10u(r)$, m/s	
2,975	4,1569	4,1569	3,025	3,1806	3,1807
2,985	4,1185	4,1185	3,035	2,7385	2,7385
2,995	4,0031	4,0031	3,045	2,2054	2,2055
3,005	3,8098	3,8098	3,055	1,5760	1,5760
3,015	3,5365	3,5365	3,065	0,8434	0,8434

Speeds in table. 2 are smaller than in table. 1, ie increasing the coefficient λ or porosity slows down the speed of the mixture.

Information on the effect of the coefficient λ on the average grain flow rate is given in table. 3, it is obtained by formula (11).

Table 3

Average grain flow rates at different λ

10λ, s/m	1	3	5	7	9
u_{av}, m/s	3,835	3,512	3,239	3,006	2,805

For comparison, in table. 4 records the speeds calculated approximately by Simpson's formula [15]:

$$u_{av} \approx \frac{1}{6r_*}\left[R_0 u(R_0) + 4u(r_*)\right], \quad (14)$$

as well as performance calculated by formula (13). According to the accepted numerical data $r_* = 0,3025$ m.

Calculations show that the value λ significantly affects the performance of the vibrating sieve Q by weight of the stair fraction.

Differences in the values of average speeds in table. 3 and table. 4 are quite small, ie Simpson's formula is quite suitable for an approximate definition u_{av}.

Conclusions. The transition to a simplified differential equation derived and tested approximate formulas for calculating the main characteristics of grain flow on a vertical cylindrical vibrating screen, taking into account the change in porosity in the layer of the grain mixture from the speed of movement. This summarizes the known theoretical results obtained using hydrodynamic models of motion of grain mixtures, fluidized by vibrations. The generalization did not significantly complicate the theory, because the final calculation formulas are quite compact and convenient in practice.

Table 4

The results of calculations by formulas (13) and (14)

10λ, s/m	1	3	5	7	9
Q, kg/s	5,210	4,360	3,702	3,183	2,765
$10u_{av}$, m/s	3,835	3,510	3,237	3,004	2,802

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ДОСЛІДЖЕННЯ ЗЕРНОПОТОКУ В ЦИЛІНДРИЧНОМУ ВІБРОРЕШЕТІ

У роботі запропоновано модифіковану гідродинамічну модель усталеного зернопотоку неоднорідної суміші на поверхні вертикального *циліндричного віброрешета. Основною гіпотезою моделі є залежність пористості суміші в рухомому кільцевому шарі від швидкості частинок. Прийнято лінійну залежність, де збільшення швидкості руху призводить до зростання пористості суміші. Для визначення швидкості зернопотоку задача зводиться до розв'язання неоднорідного диференціального рівняння типу Бесселя.*

З метою спрощення розв'язку використано метод «заморожування» змінного коефіцієнта, що допустимо через відносно малу товщину рухомого шару зерносуміші порівняно з радіусом віброрешета. В результаті такої апроксимації вдалося отримати залежність швидкості від радіальної координати у вигляді елементарних функцій. Було виведено компактну формулу для розрахунку максимальної швидкості зернопотоку, а також середньої швидкості, шляхом інтегрування відповідного виразу в елементарних функціях.

Крім того, запропоновано наближену формулу для обчислення продуктивності віброрешета за масою сходової фракції. Для цього використано метод інтегрування за формулою Сімпсона, що дозволяє уникнути складних обчислень значень спеціальних функцій для великих аргументів за допомогою асимптотичних формул. Дослідження показало, що продуктивність віброрешета суттєво залежить від пористості зерносуміші.

Для оцінки точності наближених формул було виконано чисельне інтегрування вихідного диференціального рівняння типу Бесселя на комп'ютері. Результати порівняльного аналізу підтвердили, що запропоновані спрощення мають незначні похибки і забезпечують адекватність теоретичних результатів. Таким чином, через перехід до спрощеного диференціального рівняння було розроблено та протестовано наближені формули для розрахунку основних характеристик зернопотоку на вертикальному циліндричному віброрешеті, з урахуванням зміни пористості суміші залежно від швидкості руху частинок.

Узагальнення теоретичних результатів, отриманих з використанням гідродинамічних моделей руху зерносумішей у псевдозрідженому стані під впливом вібрацій, незначно ускладнило модель, але призвело до створення компактних і практично зручних для використання розрахункових формул.

***Ключові слова**: вертикальне циліндричне віброрешето, залежність пористості від швидкості, метод «заморожування» коефіцієнта, диференціальне рівняння Бесселя, швидкість зернопотоку, продуктивність віброрешета.*

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