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## **STUDY OF THE CHARACTERISTICS OF VIBRORHEOLOGICAL MODELS OF BULK RAW MATERIALS IN THE WORKING AREA OF A VIBROCENTRIFUGAL MIXER**

*Determining the movement of loose media in vibrating equipment (in particular, vibratory volume processing, grinding, separation) is a rather complex task of dynamics, which has not been fully investigated to date. The reason for this is the cumbersomeness of the mathematical description of the dynamics of the fluid medium, simplifying hypotheses and assumptions, the linearity of the models, which do not always adequately reflect the real physics of the investigated vibrational process.*

*In recent years, vibration mechanics and vibrorheology have played an important role in the emerging new direction of applied vibration theory - the theory of vibration processes and devices. This theory examines the regularities of the impact of vibrations in various mechanical systems. It also covers the theory of machines where vibration is used for useful purposes.*

*Vibrorheology is a branch of mechanics that studies the change under the influence of vibration in the rheological properties of bodies in relation to slow forces, as well as the corresponding slow movements of bodies.*

*In recent years, vibration mechanics and vibrorheology have gained significant importance in a new direction of applied vibration theory - the theory of vibration processes and devices. This branch investigates the patterns of creation and influence of vibrations in various mechanical systems; it also covers the theory of machines where vibrations are used for useful purposes.*

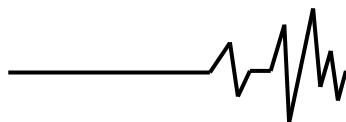
*Using the models mentioned below, it was possible to describe the chaotic movement of a layer of loose medium under the influence of vibration. These movements have been well studied in liquid media and have also been successfully studied in loose media. This further confirms the possibility of modeling the slow movements of a loose medium using analogies related to the movements of a viscous liquid.*

**Keywords:** vibration, mixing, loose medium, vibration effect, frequency, vibration mixer

**Problem formulation.** Numerical modeling has been increasingly used to describe the processes of mixing loose materials in recent decades. Numerical modeling is especially convenient when describing processes of gravity-overflow mixing, when the process occurs most actively in a thin layer of material, in which there is a random transition of component particles to the

adjacent layer. An example of such a device is a vibrating mixer (vibrocentrifugal mixer), the working cylindrical surface of which rotates together with the blades installed inside the working chamber.

Determining the movement of loose media using equipment for the implementation of vibration technologies (in particular, vibration volumetric processing, grinding, separation) is a rather



complex task of dynamics that has not been completed to date. The reasons for the latter are the unwieldiness of the mathematical apparatus for describing the dynamics of a fluid medium, simplifying hypotheses and assumptions, the linearity of models that do not always adequately reflect the real physics of the vibration process being studied [1].

**Analysis of last researches and publications.** The analysis of known studies indicates that only some scientists have investigated the effectiveness of vibration action in the separation or mixing of liquid heterogeneous systems in food or processing technologies; such studies did not have a systematic and complex nature, which causes difficulties in designing promising technological and constructive schemes of vibration processing equipment; and the developed machine designs are often characterized by high dynamic loads on the support mechanisms and an imperfect leveling system of parasitic oscillations. The fundamental advantages of vibration action as the main technological influence allows solving a number of technological problems when mixing liquid heterogeneous systems in the food industry [2].

Among high-performance technologies with relatively low energy and material costs, a significant share is those that use various vibration processes, which helps to increase the quality of processing, the level of mechanization and automation of many labor-intensive operations, economic efficiency and labor productivity, etc. Vibration processes, depending on the fields of their application, can be divided into many generalized technological directions: a) vibration effect on the

environment, bulk and dispersed systems; b) deformation and destruction; c) grinding of materials; d) the introduction of vibrating elements into environments of different densities; e) separation of the granular medium according to its geometric and other parameters. Vibrational processes are characterized by wave phenomena, and the latter, regardless of their nature and properties, are characterized by general wave patterns. Each vibration system is characterized by a drive (source of vibrations); the working space, within which the vibration energy is realized and the vibration effect on the object of processing (environment of the part, structure, etc.) is carried out [3].

**The purpose research.** An effective mathematical description of the behavior of a fluid medium under the influence of vibration is of significant applied interest. A number of interesting works are devoted to this task of mechanics, but it is still far from a satisfactory solution. In this work, based on the vibrorheological approach, some models are considered that describe slow flows of a loose medium generated by vibration

**Presentation of main material.** For the first time, the fact of "liquid" behavior of a bulk medium during vibrations was experimentally proven by Academician P. A. Rebinder and his colleagues. It has been proven that with the help of vibrations, a loose medium can be compacted and loosened, moved and bunkered, separated and mixed, as well as achieve many other useful effects. These operations are the main ones in the technological processes of feed mills, which is the reason for obtaining a positive effect from the use of vibrating feed preparation machines (fig. 1).

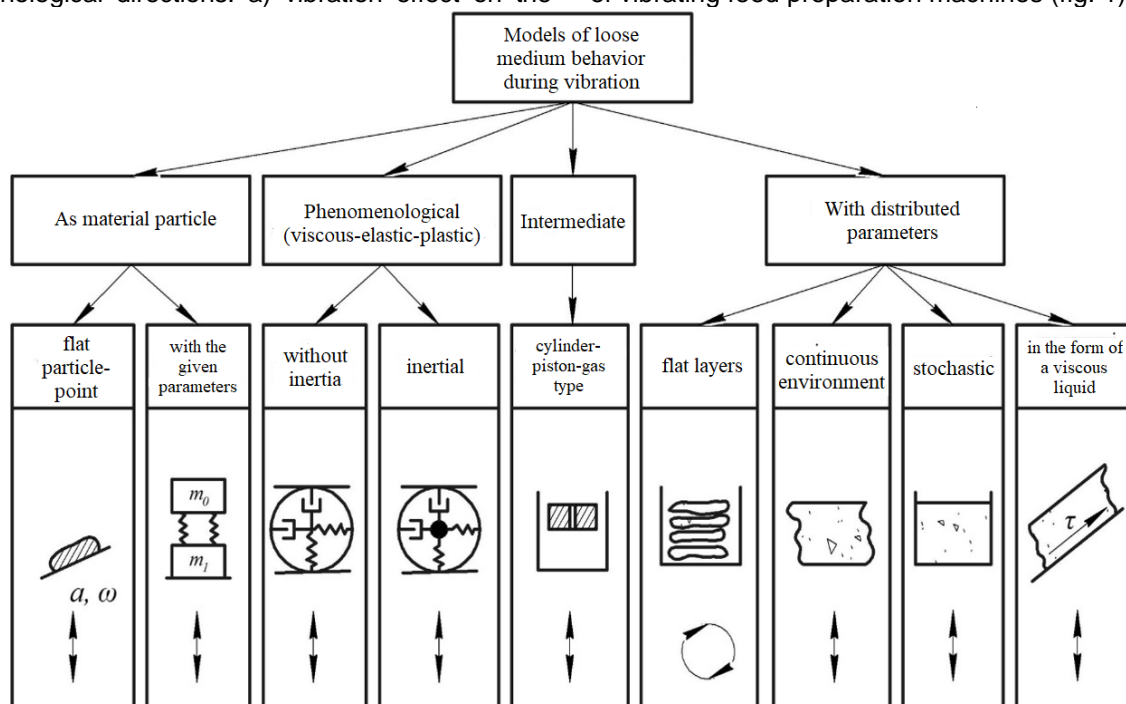
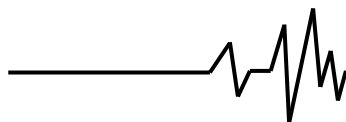


Fig. 1. Classification of existing models of behavior of loose medium



The study of the flow of highly dispersed powders during vibrations was carried out on a specially created capillary vibro-viscometer. The value of vibrational viscosity was taken as a criterion for the ability of finely dispersed powder to finish under conditions of vibrations with different parameters (frequency and amplitude) of the vibration field.

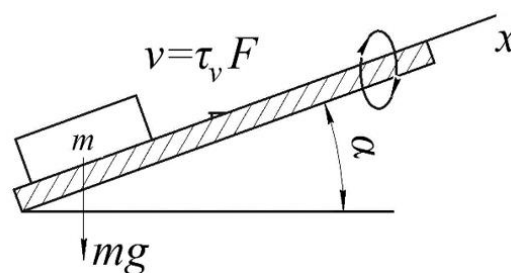
To describe the physical essence of the mixing process, it is necessary to choose two models and consider them together: a model in the form of a separate material particle and a model in the form of a viscous medium due to the complexity of the process and the lack of a single theory of mixing loose materials [4].

Let us first dwell on the description of some basic laws of the behavior of the loose medium in vibrating vessels. A certain distribution of vibration in a fluid medium in such vessels is established quite quickly - after the end of several periods of oscillations. This is a "fast" process, against the background of which slow-flowing processes take place, that is, certain flows and segregation (self-sorting) processes occur. Of course, for the study of these slow and, as a rule, the most interesting processes, the knowledge of the fast process, that is, the stable vibrational field, is of primary importance. Here we will dwell on some qualitative patterns and considerations. Let  $A$  be the amplitude, and  $\omega$  be the frequency of harmonic vibration of a vessel with a loose medium. At accelerations  $A\omega^2 < g$  for vertical  $A\omega^2 < f_1 g$  for horizontal oscillations ( $f_1$  is the coefficient of rest friction,  $g$  is the acceleration of free fall), the material moves mainly together with the vessel. At accelerations  $A\omega^2 \approx g$ , the particles of the material acquire some mutual mobility - pseudo-liquefaction begins, which leads first to compaction, and then - with a further increase in  $A\omega^2$  - to loosening and mixing. Separation processes (segregation, self-sorting) occur in the stages of pseudo-liquefaction and loosening [5].

As noted, with a layer thickness  $h$  that does not exceed a certain  $h^*$  value, and with vibration parameters that lie within certain limits, the movement of a loose medium on a vibrating flat surface can be considered as the movement of a solid body (particle). At the same time, a vibrorheological model is a body on which, along with other slow forces, the vibration force  $V$  acts. Let's call this model model **A** (fig. 2).

**A**

$h \leq h^*$

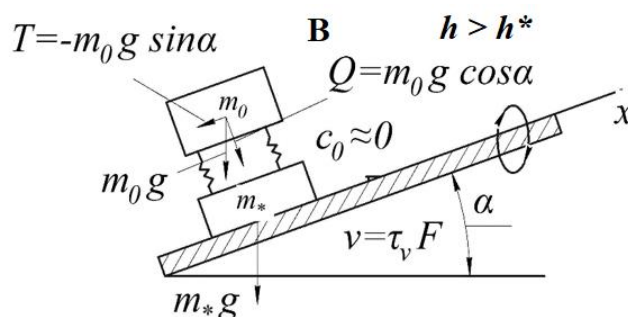


**Fig. 2. The model of the motion of a solid body (particle) at  $h \leq h^*$**

The vibration force  $V$  can be found as a result of the solution of the corresponding problem about vibration transport or using experimental data. So, for example, in the case of regimes with a fairly intense tossing of material over a flat surface performing translational oscillations along elliptical trajectories, the vibration force is determined by the approximate formula (4.53) [6]. With further consideration in mind, we note that, passing in this formula from the force  $V$  to the tangential stresses  $\tau_v = V / F$  where  $F$  is the surface area of the body that collides with the vibrating surface, it is possible to imagine the specified formula as follows:

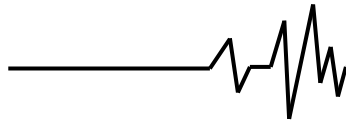
$$\tau_v = \tau_v(v_\tau) = \tau_{v0} \left[ 1 - \left( \frac{v_\tau}{v_{\tau 0}} \right) \right]. \quad (1)$$

In the case when the thickness of the layer of loose medium  $h$  exceeds the value  $h^*$  specified above, you can use a more complex model (model **B**) presented in fig. 3.



**Fig. 3. The model of the motion of a solid body (particle) at  $h > h^*$**

A body with mass  $m^*$  is in direct contact with a vibrating flat surface, connected in this case to a body with mass  $m_0$  by means of elastic elements with very low stiffness  $c_0$ ; while  $m^* + m_0 = m$ , where  $m$  is the mass of the entire layer. Thus, an additional constant force  $Q = m_0 \cdot g \cdot \cos \alpha$  acts on the body  $m^*$ ; at the same time, the same formulas as in the previous case are valid for the vibration force  $V$  and the tangential stress  $\tau_v$ , but



when  $Q = m_0 \cdot g \cdot \cos \alpha \neq 0$ . In the first approximation, we can assume:

$$m_* = \rho \cdot F \cdot h_*, m_0 = \rho \cdot F(h - h_*), (m = m_* + m_0 = \rho \cdot F \cdot h), (2)$$

$\rho$  – bulk density of the loose medium;

$h^*$  – as above, the limit thickness of the layer, at which the movement of the layer on the vibrating surface can still be considered as the movement of a solid body (particle),  $h > h^*$  – the total thickness of the layer;

$F$  – the area of the flat surface occupied by the loose medium.

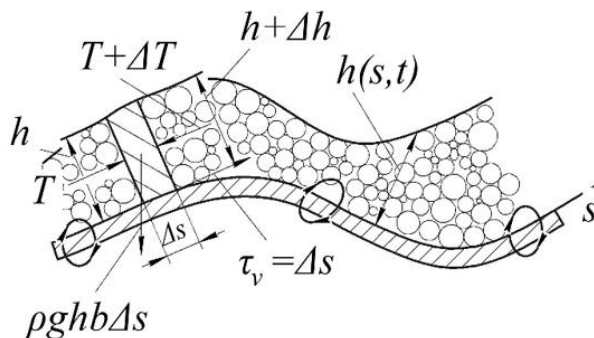
For certainty, we will assume that the values of  $\rho$ ,  $h$ , and  $h^*$  are related to the state of the medium in the absence of vibration.

The described model, despite its extreme simplicity, allows us to explain the decrease in the average speed of the layer movement, and in many cases the specific productivity, as the layer thickness  $h$  increases. This decrease is explained by the decrease as  $h$  increases in the overload parameter  $w_1$ , which is proportional to the force  $Q$ . The model also explains the increase in the "material separation phase"  $\delta_0 = \arcsin 1 / w_1$  with increasing  $h$ . In certain cases, it is advisable to refine the considered model by introducing a damped element between the masses  $m^*$  and  $m_0$ .

One of these cases is the movement of a layer of loose medium on a non-homogeneous vibrating non-flat surface, when the thickness of the layer, generally speaking, is different at different points of this surface. The corresponding task is of interest for a number of applications. Studying the one-dimensional movement of the medium in a tray with a rectangular cross-section, we assume that the points of the tray carry out periodic oscillations, the parameters of which may depend on the arc coordinate  $s$  of the surface point.

Let us first consider the case when the layer thickness  $h$  is everywhere smaller than the value  $h^*$  introduced above (fig. 4).

**A<sub>1</sub>  $h < h^*$**



**Fig. 4. The model of motion of a rigid body (particle) when  $h$  is everywhere smaller than  $h^*$**

Having drawn up the equation of motion of the layer element  $\Delta s$ , in this case we arrive at the following vibrorheological equation describing the slow ("creeping") movement of the medium layer (let's call the corresponding model model **A<sub>1</sub>**):

$$ph \frac{\partial v_\tau}{\partial t} = -\rho \cdot g \cdot h \cdot \sin \alpha - \frac{\partial T}{\partial s} + \tau_v, (3)$$

$v_\tau = v_\tau(s, t)$  – the slow component of the velocity of the medium, and  $T = T(s, t, h)$  – the force of interaction between the elements of the medium, related to the unit of "width" of the layer.

We consider the force  $T$  to be similar to the hydrostatic force acting on the side surface of the element and is determined by the formula:

$$T = \frac{1}{2} \cdot \frac{\rho \cdot g \cdot h^2}{\cos \alpha}. (4)$$

Of course, it is also assumed that the change in the inclination angle  $\alpha$  and the surface vibration parameters over a period of time of the order of the oscillation period  $T = 2\pi / \omega$  is insignificant, and therefore, when calculating the vibration stress  $\tau_v$ , the specified values can be considered as "frozen". As above, for certainty, we will relate the bulk density of the medium  $\rho$  and the height of the layer  $h$  to the state of the medium in the absence of vibration. The vibrational stress  $\tau_v$  depends on the speed  $v_\tau$ , on the layer thickness  $h$  and on the coordinate  $s$ .

It is necessary to connect the relation to equation (3):

$$\frac{\partial h}{\partial t} = -\frac{\partial(v_\tau \cdot h)}{\partial s}, (5)$$

In the case of oscillations of points along elliptical trajectories for regimes with intensive tossing, when:

$$w_1 = \frac{B \cdot \omega^2}{g \cdot \sin \alpha} > \frac{3,5(1+R)}{(1+R)^2}, (6)$$

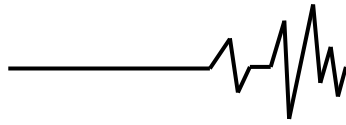
according to (1), (2), we have the following expression for the vibrational stress (we take into account that  $Q = 0$  when  $h < h^*$ ):

$$\tau_v = \rho \cdot g \cdot h \cdot q_1 \cdot \cos \alpha \left( 1 - \frac{v_\tau}{v_{\tau 0}} \right),$$

Where

$$q_1 = \frac{\lambda}{2 - \lambda} \cdot \frac{1 - R}{1 + R} \cdot \frac{A}{B} \cos \delta, v_{\tau 0} = A \omega \cdot \cos \delta. (8)$$

Taking into account expressions (4) and (7), the system of differential equations (3), (5) can be represented in the following formula:



$\frac{h}{g} \cdot \frac{\partial v_\tau}{\partial t} = -h \cdot \sin \alpha - \frac{1}{2} \cdot \frac{\partial}{\partial s} \left( \frac{h^2}{\cos} \right) + h \cdot q_1 \cdot \cos \alpha 1 - \left( \frac{v_\tau}{v_{\tau 0}} \right)$  thickness  $h(s, 0)$  and velocity  $v_\tau(s, 0)$  can be specified as initial conditions.

$$\frac{\partial h}{\partial t} = -\frac{\partial(v_\tau \cdot h)}{\partial s}. \quad (9)$$

Note that here the quantities  $\alpha$ ,  $q_1$ , and  $v_{\tau 0}$  are, generally speaking, functions of the arc coordinate  $s$ . The initial distribution of layer

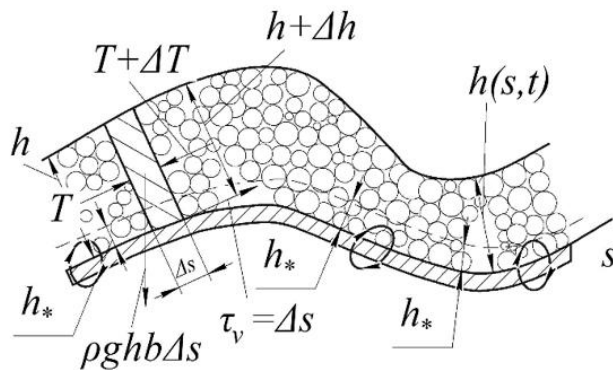
movement of a layer of loose medium under the influence of vibration is of primary interest. In this case, considering in equations (9)  $\partial v_\tau / \partial t = 0$  and  $\partial h / \partial t = 0$ , we obtain the following ordinary differential equation for the layer height  $h$ :

$$\frac{\partial h}{\partial t} = -\frac{1}{2} \sin 2\alpha - \frac{1}{2} h \cdot \cos \alpha \frac{d}{ds} \left( \frac{1}{\cos \alpha} \right) + q_1 \cdot \cos^2 \alpha \left( 1 - \frac{c}{b \cdot h \cdot v_{\tau 0}} \right), \quad (10)$$

$c = v_\tau h b = \text{const}$  denote the volume flow rate of the medium through the cross section of the tray ( $b$  is the width of the tray).

Let us now consider the case when  $h > h^*$  over the entire width of the layer. As in model **B**, we will assume that the inertia is concentrated in the lower part of the layer of thickness  $h^*$ , directly in contact with the vibrating surface, and the part located above acts on the lower one with the help of forces similar to hydrostatic pressure; the corresponding model (model **B**<sub>1</sub>) is presented in fig. 5.

**B**<sub>1</sub>  $h > h^*$



**Fig. 5. The model of motion of a rigid body (particle) when  $h$  is greater than  $h^*$  everywhere**

Due to the fact that the thickness of the "inertial part" of the layer in this case is fixed, instead of the equation of motion (3) we will have the relation:

$$ph_* \cdot \frac{\partial v_\tau}{\partial t} = -\rho \cdot g \cdot h \cdot \sin \alpha - \frac{\partial T}{\partial s} + \tau_v, \quad (11)$$

and the approximate expression for the force  $T$ , as before, is determined by formula (4).

It should be expected that models **B** (fig. 3) and **B**<sub>1</sub> (fig. 5) will be the most suitable for studying stationary movements of a layer of loose

medium, as well as in cases where the angle  $\alpha$  and vibration parameters change with the coordinate  $s$  quite slowly. Discrete analogues of models **A**<sub>1</sub> (fig. 4) and **B**<sub>1</sub> (fig. 5) were considered in work [5].

Note that models **A**<sub>1</sub> and **B**<sub>1</sub> are easily generalized to the two-dimensional case when the bulk medium moves along a non-cylindrical vibrating surface. Equations (10) and (11) in this case are replaced by the vector relation, respectively:

$$ph = \frac{\partial v_\tau}{\partial t} = -\rho \cdot g \cdot h - \text{grad} \cdot T + \tau_v, \quad (12)$$

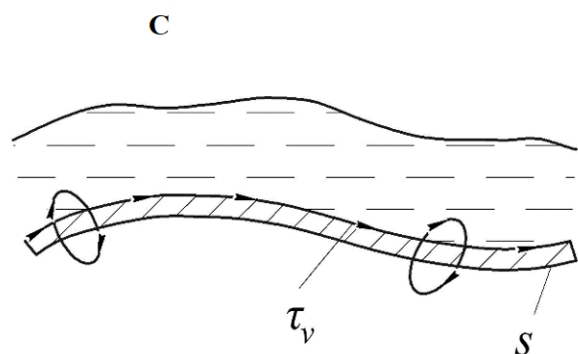
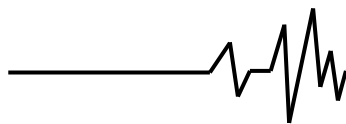
$$\frac{\partial h}{\partial t} = -\text{div}(v_\tau \cdot h). \quad (13)$$

Here, the force  $T$  is determined by formula (4), and to obtain the expressions for the vibrational stress  $\tau_v$ , the solution of the two-dimensional problem of vibration transport and separation is necessary. Separation processes under the action of gravitational and centrifugal forces are implemented only in the presence of a difference in the density or density of the elements of the dispersed and dispersed phases [7].

Along with the considered cases  $h < h^*$  and  $h > h^*$ , situations where the first inequality is fulfilled in some areas along the length of the tray, and the second inequality in others may be of interest; then it is necessary to combine the solutions of the corresponding differential equations into a pair.

The models described above do not allow studying interlayer motion in relatively thick layers of the medium, which is essential for a number of applications. Such processes can be studied on the basis of the most universal of the considered models (model **C**; fig. 6).





**Fig. 6. The movement of a loose medium, like the movement of a viscous liquid**

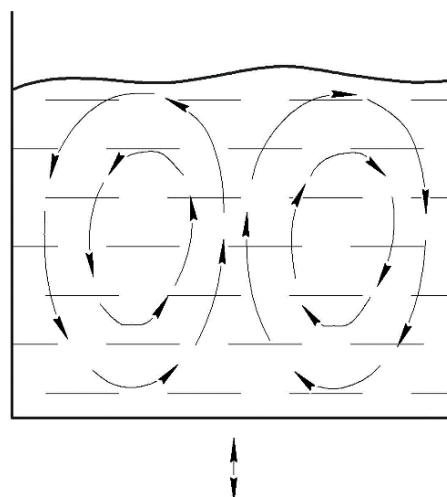
According to this model, the slow movement of a fluid medium with a fairly intense vibration of its particles is considered as the movement of a viscous (not necessarily Newtonian) liquid, the rheological characteristics of which, as well as the density, depend on the nature of the vibration. At the same time, in areas where the vibration is not intense enough and fluidization does not occur, the loose medium can be considered as a solid body; with a refined analysis, it is also possible to consider the transition zone, in which there is no fluidization of the medium, but only a decrease in the effective coefficients of dry friction. As for the conditions on the boundary  $S$ , instead of the usual sticking conditions  $v_{ls} = 0$  for a viscous liquid, as in the previously considered models, at the areas where the medium collides with the walls, the expression for the tangential stress  $v_{ls} = \tau_v(v_r)$ .

As noted, the expression for  $\tau_v(v_r)$  can be found either analytically or based on fairly simple experiments; in some cases, obtained for regimes of a certain nature can be considered as an approximation suitable for studying other regimes as well. However, as is clear from the above, the value  $m^* / \rho F = h^*$  (thickness of the layer in contact with the vibrating walls), in fact, can be considered as a certain empirically determining coefficient. Note that in some cases the dependence of  $\tau_v(v_r)$  on the value of  $m^*$  is relatively weak, and sometimes it can be neglected altogether.

As it has already been noted several times, researchers have long drawn attention to the analogy between the behavior of a liquid and a loose medium during vibration; this analogy was substantiated and theoretically proven for a rather rarefied medium of "almost elastic" particles in the work of H. I. Raskin. The inadequacy of this analogy was also noted above, in particular, the need to take vibrational forces into account when studying slow movements. This circumstance plays a primary role in the formulation of the boundary conditions, which is the main point in the above sentence (model **C**). In some cases, it may

be necessary to take into account not only surface, but also volume vibration forces in a liquid that simulates a loose medium during vibration. In particular, this is necessary when studying "slow" processes of vibrational separation (segregation) of components of loose mixtures.

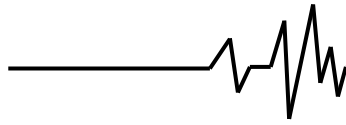
Calculation of volume vibration forces allowed A.Ya. Fidlin to explain and describe slow "convective" flows arising in a symmetrical and symmetrically vibrating vessel (fig. 7)



**Fig. 7. The movement of a fluid medium, such as a Newtonian fluid**

A. Ya. Fidlin models a loose medium in the form of a compressible Newtonian fluid with density and other parameters depending on the vibration characteristics at a given point of the medium. At the same time, it is assumed that the vibration field in the medium can be described by one scalar parameter - "vibration temperature", which, according to the theory of vibration conduction, is subject to the heat conduction equation in the presence of a heat source. In such assumptions, the role of volumetric vibrational forces is played by the input due to the dependence of the density on the vibration temperature of the Archimedean-type force.

To estimate the parameters of the proposed model, A. Ya. Fidlin considered a chain of  $N$  identical particles moving in the field of gravity over a vibrating plane and not absolutely elastically reflecting one another (fig. 8). The movement of such a chain was studied numerically on a computer, which made it possible to obtain the mentioned estimates.



**Fig. 8. Movement in the field of gravity over a vibrating plane**

Numerical experiments with a chain of particle reflections served as the basis for the physical model of the behavior of a loose medium under the influence of vibration, proposed by E. B. Kremer and A. L. Fidlin; this model (model **C**) to a certain extent can also be classified as vibrorheological. One of the main facts revealed by numerical simulations was that the collision frequency between particles was much higher than the vibration frequency. This made it possible to carry out statistical averaging of energy and momentum transfer equations using the central limit theorem of probability theory. The one-dimensional continuum model built on this basis leads to a rather complex nonlinear system of partial differential equations, which, however, can be solved analytically in some of the simplest individual cases.

Note that certain difficulties are associated with the generalization of model **C** to the three-dimensional case, as well as with the formulation of boundary conditions corresponding to this model [8].

When describing the movement of a material particle in the working body of a vibrating mixer, the following assumptions are made:

a) the circulation movement of a material particle consists of its lifting along the cylindrical surface of the chute and rolling off the free surface of the loose material, and the trajectory of the particle's movement is a twisted line;

$$F_{I \sin \beta \cos \alpha} \geq F_m - F_{mp} \cdot \sin \delta \sin \alpha + N \cos \alpha \cdot \cos \varphi_1 + F_{mm} \cos \delta \cos \alpha \sin \varphi_1 \quad (14)$$

or

$$Am\omega^2 \sin \alpha \sin \beta \cos \alpha \geq mg - fmg \cdot \sin \delta \cdot \sin \alpha + fmg \cdot \cos \alpha \cdot \cos \varphi_1 + fmg \cos \beta \cos \alpha \sin \varphi_1, \quad (15)$$

$m$  – mass of a particle of loose material, kg;

$\omega$  – circular frequency of chute oscillations,  $s^{-1}$ ;

$A$  – amplitude of chute oscillations, m;

$\beta$  – angle of direction of chute vibration, degrees;

$\alpha$  – angle of inclination of the working body to the horizon, degrees;

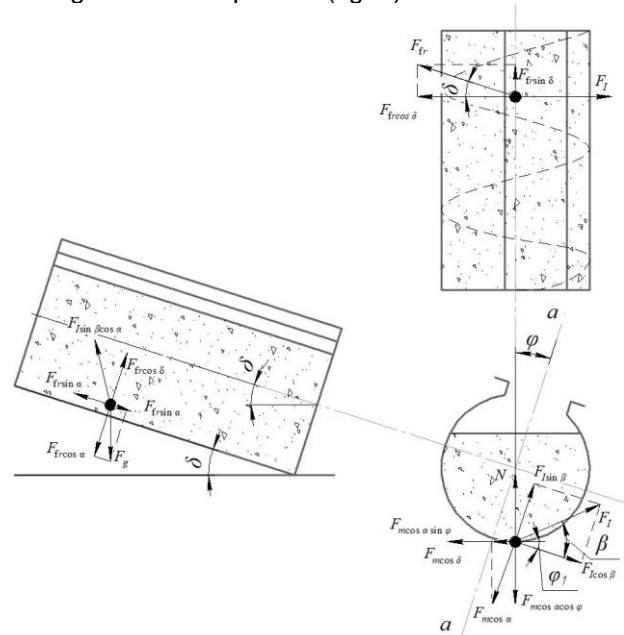
b) the force of gravity is small compared to other forces acting on the material part;

c) the angular frequency of oscillations of the trough  $A$  is constant,  $A = \text{const}$ ;

d) chute vibration angle  $P$  is constant,  $P = \text{const}$ ;

e) the angle of inclination of the mixer  $a$  is constant,  $a = \text{const}$ .

To determine the main parameters of the vibratory mixer, we present a diagram of the forces acting on the feed particle (fig. 9).



**Fig. 9. Scheme of forces acting on a particle of loose material:  $F_I$  - inertia force (vibration force);  $F_g$  - force of gravity;  $F_{fr}$  - force of friction;  $N$  - force of normal reaction;  $a$  - a vertical plane**

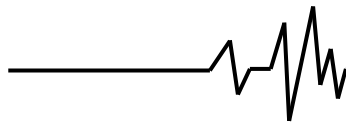
To ensure circulation movement in the state of "pseudo-liquefaction", the condition must be met: the inertial force of a particle of loose material must exceed the forces of gravity and friction. In the vertical plane, this condition looks like

$\varphi_1$  – angle of inclination of the loose material, equal to the angle of sliding friction, degrees;

$g$  – acceleration of gravity,  $m / s^2$ ;

$f$  – coefficient of internal friction.

Since in condition (15) the left part has a maximum value at  $\omega t = \pi / 2$ ,  $N = 0$ , the separation of the particle occurs when



$$F = \frac{A\omega^2 \sin \beta \cos \alpha}{g(1 - f \sin \delta \cdot \sin \alpha + f \cos \delta \cos \alpha \sin \varphi_1)} > 1, \quad (16)$$

$F$  – dimensionless parameter called the mode coefficient of the vibrating unit, which characterizes the state of the bulk medium; when  $F < 1$  – the movement of the material occurs without separation from the surface of the working body; when  $F > 1$  – by a margin; at  $F = 1$  – the limit of modes with and without separation of the particle from the vibrating surface.

When condition (14) is met, intensive mixing of components occurs in the working body, as particles of the medium begin to lose contact with the vibrating surface of the working body, and the fluid medium enters the state of "pseudo-separation" and "vibroboiling", where effective and complete destruction of its structure and rupture occur connections between particles, while the acceleration of the normal component of the particle's inertial force is greater by the value of  $g = 9.8 \text{ m/s}^2$ :

$$\frac{A\omega^2 \sin \beta \cos \alpha}{1 - f \sin \delta \cdot \sin \alpha + f \cos \delta \cos \alpha \sin \varphi_1} > g. \quad (17)$$

By substituting the values of the amplitude of oscillations, the angle of vibration and the coefficient of internal friction of the material, we calculate the minimum value of the angular frequency of oscillations when using transverse vibration

$$\omega = \sqrt{\frac{g(1 - f \sin \delta \cdot \sin \alpha + f \cos \delta \cos \alpha \sin \varphi_1)}{A \sin \beta \cos \alpha}}. \quad (18)$$

From fig. 9 it can be seen that the angle of rise of the helical line  $\delta$  in the vibrating mixer depends on the angle  $\alpha$ . The friction force of the loose material  $F_{fr}$  is directed along the tangent to the helical line. Under the action of the inertial force  $F_i$ , the feed particle moves up along the helical line. The component of the frictional force in the transverse plane is opposite in the direction of the upward movement of the particle and has the sign "-".

Without taking into account the force of inertia (vibration force)  $F_i$  when a particle rolls on a free surface, the conditions for the equilibrium of forces for its particle with mass  $m$  have the form

$$F_{fr} \sin \alpha = F_{fr} \sin \delta, \quad (19)$$

$$F_{fr} r \cos \alpha \sin \varphi_1 = F_{fr} \cos \delta, \quad (20)$$

$\varphi_1$  - angle of inclination of the loose material in the cross section to the vertical  $a-a$  (fig. 9).

From equations (17) and (18) we obtain:

$$\tan \delta = \frac{\tan \alpha}{\sin \varphi_1}. \quad (21)$$

This allows us to clarify the formula for determining the average speed of transportation of bulk material inside the working container [4].

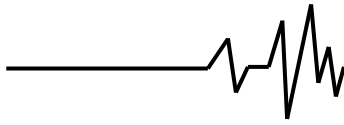
**Conclusions.** Models of the behavior of a loose medium during the action of vibration are proposed. It is based on the approach of I.I. Blechman to the analysis of equations that describe slow movements - the basic equations of vibrational mechanics.

Analytical dependences for an effective mathematical description of the behavior of a loose medium under the influence of vibration have been obtained.

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#### **ДОСЛІДЖЕННЯ ХАРАКТЕРИСТИК ВІБРОРЕОЛОГІЧНИХ МОДЕЛЕЙ СИПКОЇ СИРОВИНИ В РОБОЧІЙ ЗОНІ ВІБРОВІДЦЕНТРОВОГО ЗМІШУВАЧА**

Визначення руху сипких середовищ у вібраційному обладнанні (зокрема вібраційного об'ємного оброблення, помелу, сепарації) є доволі складним завданням динаміки, яке до сьогодні остаточно не досліджене. Причиною цього є громіздкість математичного опису динаміки сипкого середовища, спрощувальні гіпотези та припущення, лінійність моделей, що не завжди адекватно відображають реальну фізику досліджуваного вібраційного процесу.

За останні роки важливу роль в сформованому новому напрямку прикладної теорії коливань - теорії вібраційних процесів і пристроїв, відіграють вібраційна механіка і віброреологія. Ця теорія досліджує закономірності впливу вібрацій у різних механічних системах. Вона також охоплює теорію машин, де вібрація використовуються для досягнення корисних цілей.

Під віброреологією розуміють розділ механіки, в якому вивчається зміна під впливом вібрації реологічних властивостей тіл по відношенню до повільних сил, а також відповідні повільні рухи тіл.

За останні роки вібраційна механіка та віброреологія здобули суттєве значення у новому напрямку прикладної теорії коливань - теорії вібраційних процесів і пристроїв. Ця галузь досліджує закономірності створення та впливу вібрацій у різних механічних системах; вона також охоплює теорію машин, де вібрації використовуються для досягнення корисних цілей.

Використовуючи нижчезгадані моделі, вдалося описати хаотичний рух шару сипкого середовища під дією вібрації. Дані рухи були добре досліджені в рідинних середовищах, а також були успішно досліджені в сипких середовищах. Це додатково підтверджує можливість моделювання повільних рухів сипкого середовища за допомогою аналогій пов'язаних із рухами в'язкої рідини.

**Ключові слова:** вібрація, змішування, сипке середовище, вібраційний вплив, частота, вібраційний змішувач.

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