

**Paulikienė S.**Doctor of Technical Sciences,  
Professor**Vytautas Magnus  
University****Glowacki S.**Doctor of Science in Engineering,  
Professor**Warsaw University of Life  
Sciences****УДК 621.01****DOI: 10.37128/2306-8744-2026-1-8****EVALUATION OF THE ACCURACY  
OF DIMENSIONAL-  
STRENGTHENING HEAT-  
OXIDATION TREATMENT OF  
CARBIDE PRODUCTS**

*The article investigates the accuracy of dimensional strengthening thermo-oxidative treatment of hard-alloy products. The study analyzes how temperature field nonuniformity in the furnace and temperature fluctuations during processing influence the dispersion of dimensions of tungsten–cobalt alloy parts. A theoretical model describing the relationship between dimensional deviations before and after thermo-oxidation is developed using statistical methods and the theory of random processes.*

*The influence of constant temperature gradients and random temperature fluctuations on the oxidation depth and dimensional accuracy is analytically evaluated. Correlation functions of temperature variations in furnaces with different control systems are determined, and their effect on the variance of the removed material layer during thermo-oxidation is established. Experimental studies on hard-alloy samples confirm the theoretical dependences and demonstrate that even small temperature differences in the furnace can lead to noticeable dimensional deviations.*

*The research also examines the effect of oxidation conditions (air and oxygen atmosphere) on the dimensional accuracy of hard-alloy elements used in high-pressure chambers. Results show that thermo-oxidative treatment improves surface microrelief and can slightly reduce the dispersion of dimensions. It is established that increasing the oxidation time while decreasing the oxidation rate and furnace temperature can significantly reduce dimensional variance and improve processing accuracy.*

*The obtained results provide recommendations for optimizing thermo-oxidative processing parameters and improving dimensional accuracy in the manufacturing of hard-alloy components.*

**Key words:** thermo-oxidative treatment, hard alloys, tungsten–cobalt alloys, dimensional accuracy, temperature fluctuations, oxidation kinetics, dimensional dispersion, temperature field nonuniformity, oxidation depth, surface microrelief.

EVALUATION OF THE ACCURACY OF DIMENSIONAL-STRENGTHENING HEAT-OXIDATION TREATMENT OF CARBIDE PRODUCTS © 2026 by Paulikienė Simona, Glowacki Szymon is licensed under CC BY 4.0

**Problem formulation.** Thermo-oxidative treatment (TOT) is widely used in the technological processing of hard-alloy products as a method of dimensional strengthening and finishing. This process is based on controlled high-temperature oxidation of the material surface, followed by the removal of the formed oxide layer. Due to this mechanism, thermo-oxidative treatment allows the formation of required geometrical parameters, improvement of surface quality, and stabilization of the structural and mechanical properties of hard

alloys. The method is particularly important in the manufacturing of precision components made of tungsten–cobalt alloys, which are widely applied in high-pressure equipment, cutting tools, and wear-resistant elements.

Despite its technological advantages, ensuring high dimensional accuracy during thermo-oxidative treatment remains a complex problem. Unlike conventional mechanical processing methods, the material removal during thermo-oxidation depends not only on geometric



parameters and tool positioning but also on physicochemical processes occurring at elevated temperatures. The oxidation rate is strongly influenced by temperature, oxidation environment, exposure time, and the microstructure of the material. As a result, even small deviations in technological parameters may lead to significant variations in the thickness of the removed layer and, consequently, in the final dimensions of the treated parts.

One of the most important factors affecting the stability of thermo-oxidative processing is the temperature regime in the furnace. In practical industrial and laboratory conditions, the temperature field in the working zone of a furnace is rarely perfectly uniform. Spatial temperature gradients arise due to design features of heating elements, heat losses, gas circulation, and loading conditions of the furnace chamber. Parts located in different regions of the working zone may therefore be exposed to slightly different temperatures during processing. Since the oxidation rate depends exponentially on temperature, even minor temperature differences can significantly affect the kinetics of oxidation and the resulting dimensional changes.

Another source of dimensional inaccuracy is the presence of time-dependent temperature fluctuations during the processing cycle. These fluctuations are caused by the limitations of temperature control systems, the thermal inertia of the furnace and thermocouples, variations in electrical supply voltage, and environmental conditions. As a result, the actual temperature inside the furnace can be represented as a stochastic process that continuously deviates from the nominal setpoint. Such random variations influence the oxidation rate over time and lead to random deviations in the thickness of the oxidized layer formed on the surface of the parts.

The combined effect of spatial temperature nonuniformity and temporal temperature fluctuations results in a dispersion of dimensions within a batch of processed products. Even when the initial dimensions of the parts are identical or very close, differences in thermal conditions during treatment may cause uneven oxidation depths. Consequently, the final geometrical parameters of the products may vary beyond acceptable tolerance limits. This issue becomes particularly critical in the production of precision components, where dimensional deviations of only a few micrometers may significantly affect the performance of the final product.

In addition to thermal factors, the dimensional accuracy after thermo-oxidative treatment is influenced by the characteristics of the initial surface and the microrelief of the material. Hard-alloy products obtained after sintering often possess surface irregularities, adhered particles, and inclusions of large carbide grains. These microstructural features can affect the local

oxidation rate and introduce additional variability in the thickness of the removed material layer. During thermo-oxidation, protruding micro-irregularities tend to oxidize more intensively, which may partially smooth the surface but also influence dimensional measurements.

Another important technological factor is the oxidation environment. The process may be performed in air, oxygen, or other oxidizing atmospheres. Each environment is characterized by different oxidation kinetics, which directly affects the rate of oxide layer formation and the overall depth of material removal. Therefore, the selection of oxidation medium and processing parameters must be carefully optimized to achieve both the required material removal and acceptable dimensional accuracy.

The technological regime of thermo-oxidative treatment, including processing time and furnace temperature, also plays a critical role in determining the dispersion of dimensions. For a given required oxidation depth, different combinations of temperature and time can be applied. For example, a higher temperature combined with shorter exposure time or a lower temperature combined with longer exposure time may produce the same nominal oxidation depth. However, these regimes may lead to different levels of dimensional dispersion due to the sensitivity of the oxidation rate to temperature variations.

From a technological standpoint, it is therefore necessary to develop methods for evaluating and predicting the influence of temperature instability and other process parameters on the accuracy of dimensional processing during thermo-oxidation. Such an analysis requires the application of statistical approaches and the theory of random processes, which make it possible to describe temperature fluctuations and their influence on oxidation kinetics in a quantitative manner.

Despite the practical importance of thermo-oxidative treatment in the manufacturing of hard-alloy components, the relationship between temperature fluctuations, oxidation kinetics, and dimensional accuracy has not been sufficiently studied. In particular, there is a need for a comprehensive theoretical and experimental analysis that would allow the determination of the main factors influencing dimensional dispersion and the development of recommendations for improving the accuracy of thermo-oxidative processing.

Therefore, the main problem addressed in this study is the investigation of the influence of temperature field nonuniformity, random temperature fluctuations, oxidation environment, and technological parameters of thermo-oxidative treatment on the dimensional accuracy of hard-alloy products. The research aims to develop a theoretical model describing the formation of dimensional deviations during thermo-oxidation, to



verify the model experimentally, and to identify technological conditions that ensure minimal dimensional dispersion of processed parts.

The results of such research are important for improving technological processes used in the production of precision hard-alloy components, increasing the reliability of thermo-oxidative dimensional processing, and ensuring stable quality of manufactured products.

**Analysis of recent research and publications.** Hard alloys based on tungsten carbide with cobalt binder (WC-Co) are among the most widely used engineering materials due to their high hardness, wear resistance, and thermal stability. They are extensively applied in cutting tools, mining equipment, and components operating under extreme conditions. Modern research in this field focuses on the physicochemical processes that occur during high-temperature treatment of these materials, including oxidation kinetics, phase transformations, and the influence of technological parameters on structural and dimensional stability.

A significant number of studies are devoted to the oxidation behavior of WC-based hard alloys at elevated temperatures. Researchers have shown that the oxidation process in WC-Co alloys occurs through complex chemical reactions involving tungsten carbide and the cobalt binder, resulting in the formation of oxide compounds such as  $WO_3$ ,  $CoO$ ,  $Co_3O_4$ , and complex oxides like  $CoWO_4$ . The oxidation resistance of such alloys depends strongly on their phase composition, microstructure, and the protective properties of the oxide layer formed during heating.

Experimental investigations of high-temperature oxidation in cemented carbides indicate that oxidation kinetics strongly depend on temperature and environmental conditions. Studies show that the oxidation rate increases significantly with temperature in the range of approximately 400–800 °C and may vary depending on the atmosphere (for example, dry air or water vapor). The oxide layer formed during the process may contain pores, cracks, and volatile oxide phases, which influence the stability and mechanical properties of the treated material.

Another important direction of research concerns the kinetics of oxidation processes in tungsten-based materials. It has been established that oxidation kinetics can demonstrate complex behavior, including changes in activation energy and deviations from classical parabolic oxidation laws within certain temperature ranges. For example, studies of WC-based carbides have revealed anomalous behavior in the oxidation rate between approximately 528 °C and 630 °C, where the rate may decrease with increasing temperature due to changes in reaction mechanisms and oxide scale formation.

In recent years, considerable attention has also been paid to the influence of alloy composition and binder phases on oxidation resistance.

Research indicates that modifications of the binder composition, including the introduction of additional alloying elements or high-entropy binders, can significantly improve oxidation resistance and modify the mechanism of oxide layer formation. Such approaches are used to enhance the performance of hard alloys in high-temperature environments.

Along with chemical composition, the technological parameters of thermal treatment play a crucial role in determining the oxidation behavior of hard alloys. These parameters include the temperature regime, processing time, and oxidation atmosphere. Variations in these factors influence the rate of oxide formation, diffusion processes in the material, and the thickness and structure of the oxide layer. Therefore, optimization of thermal processing regimes remains an important task in the development of advanced technologies for hard-alloy components.

Another important research direction is the study of the influence of microstructure and surface condition on oxidation processes. The morphology of the surface, grain size distribution, and the presence of defects or inclusions can significantly affect diffusion processes and chemical reactions occurring during oxidation. Consequently, the initial state of the material and its surface preparation may influence the uniformity of oxidation and the resulting dimensional changes in processed parts.

Despite the considerable amount of research devoted to oxidation processes in tungsten-based alloys, most studies focus primarily on chemical mechanisms, phase transformations, and oxidation resistance of materials. Much less attention has been paid to the technological aspects of thermo-oxidative treatment as a dimensional processing method, particularly to the influence of temperature instability and furnace operating conditions on the dimensional accuracy of hard-alloy products.

Therefore, further research is required to investigate the relationship between temperature fluctuations, oxidation kinetics, and dimensional deviations during thermo-oxidative treatment. A comprehensive theoretical and experimental analysis of these factors would contribute to improving the technological reliability of thermo-oxidative processing and ensuring higher dimensional accuracy of hard-alloy components.

**The purpose of the article.** The purpose of this article is to investigate the influence of temperature field nonuniformity and temperature fluctuations during thermo-oxidative treatment on the dimensional accuracy of hard-alloy products, as well as to determine the technological conditions that minimize dimensional dispersion during processing.

**Results of the researches.** In the experimental study of the kinetics of high-temperature oxidation (HTO) of tungsten-cobalt alloys, it was found that the heterogeneity of the temperature field in the working zone of the furnace



and the nature of temperature fluctuations throughout the entire processing cycle directly affect the dispersion of the sizes of oxidized samples. If we denote by  $\Delta h_n$  and  $\Delta h$  the random deviations of the sizes of parts before and after TO, and by  $\Delta h_{to}$  the random deviations of the size of the removed layer during TO, then the following relation occurs:

$$\Delta h = \Delta h_n + \Delta h_{to}. \quad (1)$$

We assume that the deviations  $\Delta h_n$  and  $\Delta h_{to}$  are not correlated. Based on the theorem on the dispersion of the sum of random variables, the dispersion of the sizes (standard deviation) of the parts after maintenance will be equal to:

$$\sigma_h^2 = \sigma_n^2 + \sigma_{to}^2. \quad (2)$$

where  $\sigma_n$  is the standard deviation of the dimensions of the parts before processing;

$\sigma_{to}$  is the standard deviation of the dimensions of the parts caused by maintenance.

If, before the maintenance operation, a batch of hard alloy products is divided into groups with similar dimensions and for each group the appropriate oxidation mode is determined, the size spread of the entire batch will decrease and will be equal to:

$$\sigma_h^2 \approx (\sigma_n / n)^2 + \sigma_{to}^2. \quad (3)$$

where  $n$  is the number of size groups.

Exact equality will be in the case if the random variables of sizes have a uniform density distribution. It should be noted that the number  $n$  should be chosen such that the condition is fulfilled

$$\frac{\sigma_n}{n} \leq \sigma_{to}.$$

The influence of a constant temperature difference on the dimensional variation during simultaneous processing of a batch of carbide products was analytically evaluated as follows.

Suppose it is necessary to oxidize a batch of products of the same size to a depth  $h$ . We use the equation that relates the depth of oxidation to the oxidation rate  $k$  and the processing time  $t$ :

$$h = kt \quad (4)$$

The coefficient  $k$  depends on the temperature  $T$  exponentially:

$$k = A \exp\left(-\frac{Q}{RT}\right) \quad (5)$$

Differentiate equations (4) and (5):

$$\frac{dk}{dT} = A \exp\left(-\frac{Q}{RT}\right) \frac{Q}{RT^2} = k \frac{Q}{RT^2} \quad (6)$$

$$\frac{dh}{dt} = \frac{dk}{dt} t \quad (7)$$

$$dK = K \frac{Q}{RT^2} dT \quad \text{и} \quad dK = dh \cdot t^{-1} \quad (8)$$

Equating the right-hand sides of equations (8) and proceeding to finite increments, we obtain:

$$\begin{aligned} dh &= \frac{dh}{dt} dT = k \frac{Q}{RT^2} t dT \\ \Delta h &= \frac{kQt}{RT^2} \Delta T \end{aligned} \quad (9)$$

Expression (9) relates  $\Delta h$  — the value by which one part of a batch of parts will be oxidized

more than the other, if the temperature difference between them during the entire processing is equal to  $\Delta T$ .

On two batches of cylindrical carbide teeth of the VK8 brand, having the same outer diameter  $\varnothing 12 + 0.008 / 0$ , an experimental check of the dependence (9) was carried out. The first batch of teeth underwent oxidation at  $T_n = 900^\circ\text{C}$ , the second — at  $T_n = 909^\circ\text{C}$ . Oxidation time — 35 min. Nominal oxidation depth  $h = 150 \mu\text{m}$ . Oxidizing medium — air. Temperature control accuracy  $\pm 0.5^\circ\text{C}$ .

Measurement of the diameters of samples from both batches after oxidation showed that the difference in the average diameters between them is approximately 15 microns, i.e. a constant temperature difference of 1% from the set value during the processing leads to a difference in size values within the batch of approximately 10%.

In addition to the error introduced by the inhomogeneity of the temperature field in the furnace, the accuracy of temperature control also affects the size spread during maintenance. From the analysis of the nature and dynamics of temperature fluctuations in the furnace caused by the imperfection of the control system, it can be noted that its deviations from the set value are continuous and random.

The amplitude and frequency of oscillations are affected by the inertia of the control system and thermocouples, voltage fluctuations in the network, ambient temperature, etc. Therefore, the measurement of the reaction temperature during maintenance can be represented as a random function of time  $T(t)$ .

Let us highlight the mathematical expectation of this random process:

$$T_0 = M \cdot [T(t)]$$

Usually, during TO, the  $T_0 = \text{const}$  mode is maintained. Let us expand function (5) in the vicinity of  $T = T_0$  into a Taylor series:

$$k(T) \approx k_0 + \left(\frac{\partial k}{\partial T}\right)_{T_0} \Delta T = k_0 + \frac{k_0 Q}{RT_0^2} \Delta T \quad (10)$$

where  $k_0 = A \exp\left(-\frac{Q}{RT_0}\right)$ .

Substituting (5) into equation (4):

$$h = \int_0^t k d\tau = \int_0^t A \exp\left(-\frac{Q}{RT}\right) d\tau$$

we will get

$$h(t)h_0(t) + \Delta h(t) \quad (11)$$

where the mathematical expectation of the value of the removed layer is

$$h_0(t) = \int_0^t k_0 d\tau = k_0 t$$

The centered random process  $Dh(t)$  is related to the centered random process  $DT(t)$  by a linear homogeneous transformation:

$$\Delta h(t) = (k_0 Q / (R T_0^2)) \int_0^t \Delta T(\tau) d\tau \quad (12)$$

According to the theory of stationary random functions, it is established: if the process-



function  $\Delta h(t)$  is connected with the process-argument  $\Delta T(t)$  by a linear homogeneous transformation, then to find the correlation function of the process-function  $Kh(t, t')$  it is necessary to apply this transformation twice to the correlation function of the process-argument  $KT(t, t')$ . Therefore, based on (12), we have:

$$Kh(t, t') = (k_0 Q / (R T_0))^2 \int_{\rho^t}^{\rho^t} \int_{\rho^{t'}}^{\rho^{t'}} KT(\tau, \tau') d\tau d\tau' \quad (13)$$

First, we find the correlation function  $KT(t, t')$  of the random process  $T(t)$ .

A review of various implementations of the random process  $T(t)$  shows that over a long interval during thermal oxidation, the probabilistic characteristics of the process do not change with any shift of the arguments along the  $t$  axis. Therefore, the process  $T(t)$  is stationary.

$$T_0 = M[T(t)] = const$$

$$KT(t, t') = M[DT(t)DT(t')] = KT(t - t')$$

In addition, the process  $T(t)$  is considered ergodic for physical reasons, which allows us to

determine its random characteristics for one implementation.

The experiments were carried out on laboratory installations. The deviation of the temperature from the set point was recorded by a control potentiometer PP-63 (class 0.05) every 30 s for 150 min. The temperature in the furnaces in all experiments was 900°C.

Table 2 shows the results of statistical data processing. The course of the function  $KT(t)$  for the first and third furnaces is shown in Fig. 1 (a, b).

Analysis of the experimentally found correlation functions shows that they can be described by the expression:

$$K_T(\tau - \tau') = De^{-\alpha(\tau - \tau')} \cos \beta(\tau - \tau') \quad (14)$$

Approximation of function (14) by the least squares method gives a system of equations for determining the parameters  $\alpha$  and  $\beta$ .

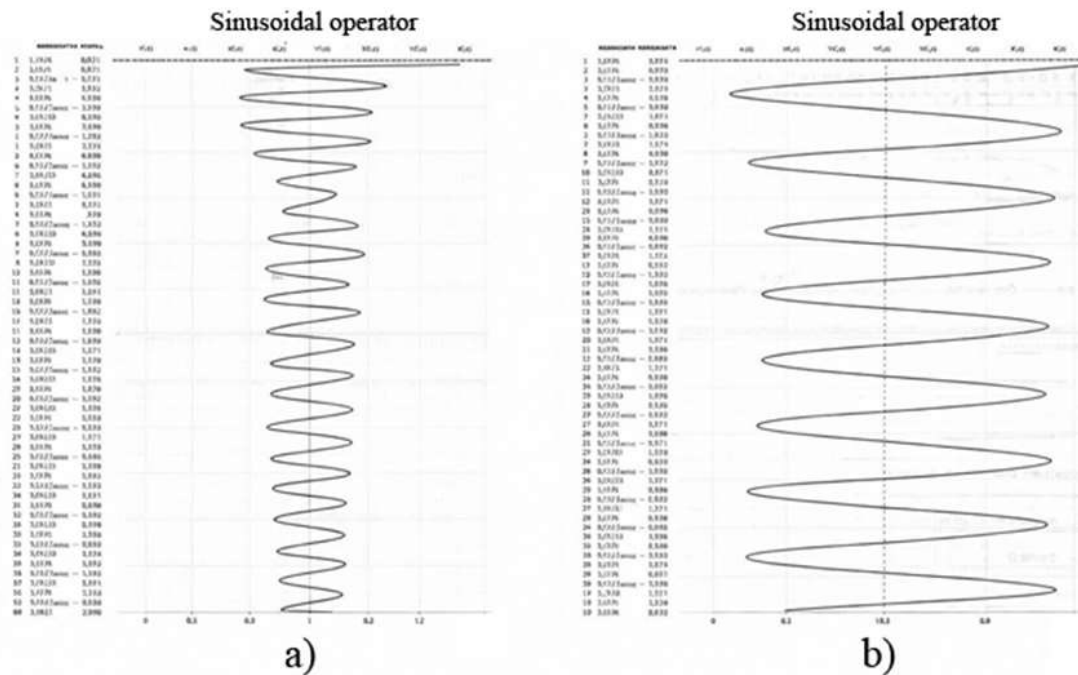


Fig. 1. The course of the obtained empirical correlation function  $K_T(t, t')$  for furnaces with different thermal control systems: a — furnace No. 3, b — furnace No. 1 (Table 2.1)

Table 1.

Characteristics of the installations used in the studies

No.	Oven type	Temperature control method	Adjustment accuracy
1	Laboratory muffle electric furnace MP-2UM	Two-position regulation with dilatometric thermostat	±14 °C
2	Electric resistance furnace for mine laboratory SSHOL-1, 1.6/12	Automatic regulation by three series-connected thermocouples PR-30/6 and millivoltmeter MR-1-02M	±10°C
3	Electric resistance furnace mine laboratory SSHOL-1, 16/12	Electronic automatic adjustment with potentiometer PSR-1-02	±4.5°C



Table 2.

Results of solving the system of equations (15)

Variables	Oven No. 1	Oven No. 2
Standard deviation	16.9°	9.33°
Dispersion D	285 °²	87.2 °²
Damping rate α	0.00555 s <sup>-1</sup>	0.0134 s <sup>-1</sup>
Oscillation frequency β	0.0572 rad/s	0.0282 rad/s
Parameter φ	84.5°	64.6°

$$\begin{cases} \sum_{i=1}^n (K_i - De^{-\alpha\tau_i} \cos \beta\tau_i) e^{-\alpha\tau_i} \cos \beta\tau_i = 0 \\ \sum_{i=1}^n (K_i - De^{-\alpha\tau_i} \cos \beta\tau_i) \tau_i e^{-\alpha\tau_i} \cos \beta\tau_i = 0 \\ \sum_{i=1}^n (K_i - De^{-\alpha\tau_i} \cos \beta\tau_i) \tau_i e^{-\alpha\tau_i} \sin \beta\tau_i = 0 \end{cases} \quad (15)$$

We find the correlation function  $K_1(t, t')$  of the random process  $Ah(t)$ , which coincides with the correlation function of the random process  $h(t)$ . To do this, we substitute (14) into (13).

$$K_h(t, t') = \left(\frac{k_0 Q}{RT_0^2}\right)^2 \times D \left[ 2 \int_0^t \int_0^{t'} e^{-\alpha(\tau-\tau')} \cos \beta(\tau-\tau') d\tau d\tau' \right] \quad (16)$$

In Figure 2, the OSCE rectangle indicates the integration region when  $t \geq t'$ .

Given the parity of the integrand with respect to the difference  $(tt')$ , the integral over the domain ODE is equal to the integral over the

Therefore, we replace equation (16) (at  $t \geq t'$ ) with:

$$K_h(t, t') = \left(\frac{k_0 Q}{RT_0^2}\right)^2 \times D \left[ 2 \int_0^t \int_0^{t'} e^{-\alpha(\tau-\tau')} \cos \beta(\tau-\tau') d\tau d\tau' + \int_0^t \int_0^t e^{-\alpha(\tau-\tau')} \cos \beta(\tau-\tau') d\tau d\tau' \right] \quad (18)$$

After integrating and introducing the notation  $\varphi = \arctg \frac{\beta}{\alpha}$ , we arrive at the equation.

domain ODA. But in the domain ODA, as well as in the domain ABCD, the integrand has the form:

$$e^{-\alpha(\tau-\tau')} \cos \beta(\tau-\tau') \quad (17)$$

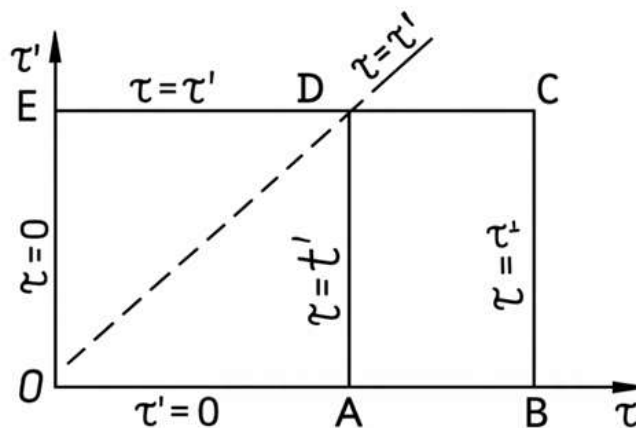


Fig. 2. Integration region for  $t \geq t'$

$$K_h(t, t') = \frac{2Dk_0^2 Q^2}{(\alpha^2 + \beta^2)R^2 T_0^4} \times \left\{ t' \sqrt{\alpha^2 + \beta^2} \cos \varphi - e^{\alpha t} \cos(2\varphi + \beta t) - e^{-\alpha(t-t')} \cos[2\varphi + \beta(t-t')] \right\} \quad (19)$$

It is easy to verify that when  $t \geq t'$  the correlation function is equal to the expression:



$$K_h(t, t') = \frac{2Dk_0^2 Q^2}{(\alpha^2 + \beta^2)R^2 T_0^4} \times \left\{ t\sqrt{\alpha^2 + \beta^2} \cos \varphi - e^{\alpha t} \cos(2\varphi + \beta t') - e^{-\alpha(t-t')} \cos[2\varphi + \beta(t-t')] \right\} \quad (20)$$

Combining these two expressions, we can write: (21)

$$K_h(t, t') = \frac{2Dk_0^2 Q^2}{(\alpha^2 + \beta^2)R^2 T_0^4} \times \left\{ \min(t, t')\sqrt{\alpha^2 + \beta^2} \cos \varphi - e^{\alpha \max(t, t')} \times \cos[2\varphi + \beta_{\max}(t, t')] e^{-\alpha(t, t')} \cos(2\varphi + \beta(t-t')) \right\} \quad (21)$$

Equating  $t = t'$  we find the dispersion of the removed layer for a duration of thermal oxidation equal to  $t$ .

$$D_n(t) = K_h(t, t') = \frac{2Dk_0^2 Q^2}{(\alpha^2 + \beta^2)R^2 T_0^4} \times \left[ t\sqrt{\alpha^2 + \beta^2} \cos \varphi + e^{\alpha t} (2\varphi + \beta t) - \cos 2\varphi \right] \\ = \frac{2DA^2 Q^2 \exp\left(-\frac{2Q}{RT_0}\right)}{(\alpha^2 + \beta^2)R^2 T_0^4} \times \left[ t\sqrt{\alpha^2 + \beta^2} \cos \varphi + e^{-\alpha t} (2\varphi + \beta t) - \cos 2\varphi \right]. \quad (22)$$

The influence of oxidation modes on the size dispersion was experimentally investigated on hard alloy elements for high-pressure chambers. The dimensions of the well — diameter  $D$  and depth  $h$  — were measured before and after heat treatment (HT). Since these parameters determine the working volume of the well, which is of the greatest importance during synthesis, the working surface of the well is not subjected to any mechanical treatment in the existing technological process for manufacturing high-pressure chambers. The size measurements were carried out with an

instrumental microscope. The measurement error is  $\pm 1 \mu\text{m}$ . The number of parts in the measured batches  $n = 100$ . Oxidation was carried out on 5 parts on stands made of Kh18N9T alloy.

The oxidation time was recorded with a stopwatch. The results of the experiments and the oxidation conditions are given in Table 3. Figure 3 a, b presents histograms and plots the normal distribution curves of the deviations in the sizes (diameter  $D$  and depth  $h$ ) of the holes in the carbide matrices before and after thermal oxidation (in oxygen and in air).

**Table 3**

**The influence of oxidation modes on the accuracy of dimensional processing**

Oxidation modes $T_h = 900^\circ\text{C}$ oven number 2	Diameter, mm	Depth, mm
Before processing	$19.65 \pm 0.203$	$5.46 \pm 0.085$
Maintenance in air (t = 30) min	$19.76 \pm 0.192$	$5.39 \pm 0.108$
TO in oxygen environment (t = 8) min	$19.77 \pm 0.183$	$5.39 \pm 0.084$

It is assumed that the dispersion of the measured batch sizes follows a normal

$$\text{distribution: } Y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(X_{ch}-\sigma)^2}{2\sigma^2}}$$

The deviations in size were taken equal  $3\sigma$  where  $\sigma$  is the standard deviation of the batch size,

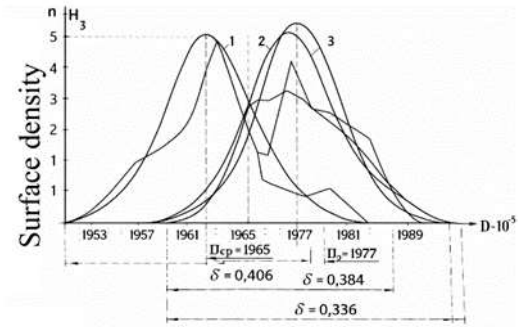
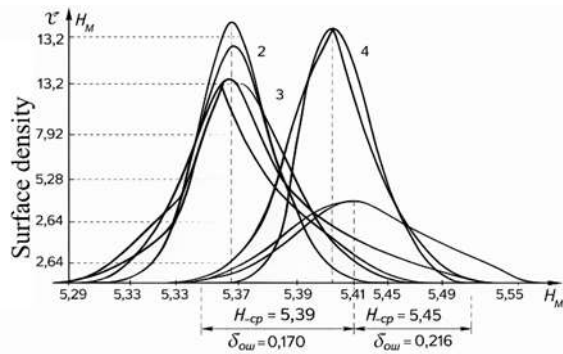
$X_{cp}$  - average size.

Both in air (Fig. 3, curve 2a) and in oxygen (Fig. 3, curve 2b) slightly reduced the value of the mean square deviations of dimensions when removing the allowance by 0.11–0.12 mm. The best results were obtained with maintenance in oxygen, although the histograms of the distribution of  $D$  and

$h$  after maintenance became lower and flatter, i.e. a decrease in the number of parts in the batch with sizes close to the average is clearly observed.

A satisfactory explanation may be the fact that as a result of the treatment in air and oxygen, the surface microrelief significantly improved. The average height of microroughnesses decreased from  $3.4 \mu\text{m}$  to  $0.60 \mu\text{m}$ .

As is known [3], the surface relief of the product after sintering consists of adhered particles of backfill, inclusions of large grains of tungsten carbide, depressions and other irregularities, which introduce a significant error in determining the size of the product.



**Fig. 3. The influence of oxidation modes (alloy VK-6) on the accuracy of dimensional processing: 1 — variance of the initial batch sizes; 2 — size dispersion after thermal oxidation in air; 3 — size dispersion after thermal oxidation in an oxygen environment a — data obtained when measuring the depth of the hole in high-pressure chambers;b — data obtained when measuring the diameter of the hole.**

During thermal oxidation, significant heat is released, which contributes to more intensive oxidation of microprotrusions.

Reducing the height of micro-unevennesses in combination with a relatively small allowance (0.2 mm) obviously allows to compensate for the negative impact of temperature fluctuations on the size spread, especially since the oxidation is carried out in a furnace with high precision temperature control (furnace No. 3, Table 1) and in an isothermal field.

However, when simultaneously processing products with a small height of micro-unevenness ( $Ra = 0,32 - 0,16 \text{ mkm}$  and lower) without prior division into size groups, it is impossible to reduce the size dispersion of the entire batch due to the lack of active control of the thickness of the oxidized layer on each product.

The study of the reliability of equation (22) and the influence of the parameters included in this equation on the size spread was carried out on ground cylindrical carbide teeth.  $\varnothing 12_0^{+0,15} X14$  brand **VK8V** when developing the technology for obtaining a chamfer for pressing.

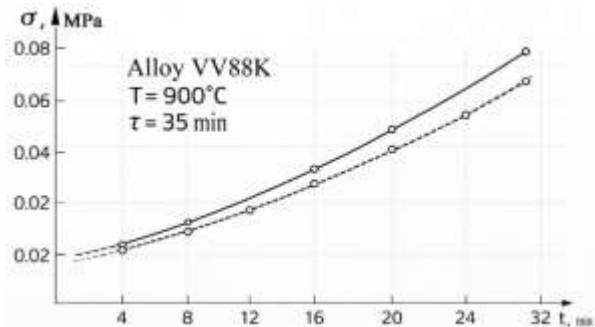
The total number of teeth (1000 pcs.) was divided into groups of 20 pcs. with the same size. Within each group, the deviations in height from the average did not exceed  $0.5 \mu\text{m}$ .

The oxidation modes, which obtained the desired chamfer profile, were the same in all cases. Processing time  $t = 30 \text{ min}$ , furnace temperature  $T_n = 900^\circ\text{C}$ , oxidizing medium — air. The difference was that each group was oxidized in furnaces with different temperature control systems.

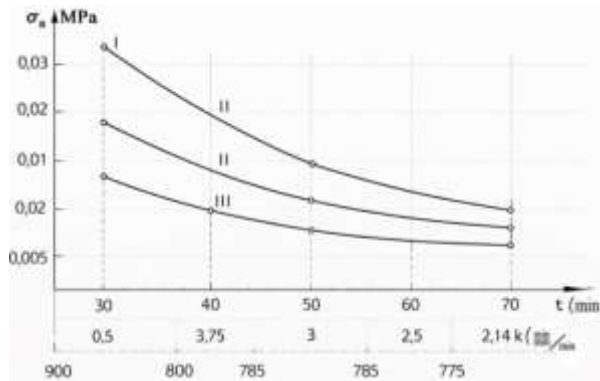
After oxidation and short-term wet treatment (to remove oxides), the size of the teeth was measured and the root mean square deviations of the height  $\sigma_h$  were calculated. Theoretically calculated according to equation (24) the values of  $\sigma_h$  (Fig. 4, curve 2) are close to the experimental ones (curve 1). At the same time, a slightly larger actual value of  $\sigma_h$  is observed, which is explained

by the presence of impurities in the alloy, their non-uniform distribution in the teeth, the effect on the oxidation rate, and also the practically unequal holding time in the furnace. As can be seen from Fig. 4, the shorter the oxidation time and the smaller the depth of the oxidized layer, the smaller the size spread.

Figure 5 shows the experimentally obtained dependence of the root mean square deviation of the dimensions  $\sigma_{ah}$  on the process parameters of the oxidation rate  $k$ , time  $t$ , and furnace temperature  $T_n$ . The oxidation conditions were chosen such that the thickness of the oxidized layer was always the same (0.15 mm). With an increase in time  $t$  and a corresponding decrease in the oxidation rate  $k$  and furnace temperature  $T_n$ , the dispersion of the dimensions  $\sigma_h$  significantly decreases. Therefore, one of the ways to achieve the required machining accuracy can be to increase the oxidation time, which can lead to an increase in machining accuracy. High machining accuracy (2nd accuracy class) can be achieved even in furnaces with an imperfect temperature control system.



**Fig. 4. The influence of oxidation time on the standard deviation of the dimensions of oxidized parts: 1 — theoretically obtained curve according to equation (22); 2 — experimentally obtained curve (furnace No. 3, Table 1).**



**Fig. 5. Dependence of standard deviations on process parameters - oxidation rate, oxidation time, furnace temperature 1 - furnace No. 1, 2 - furnace No. 2, 3 - furnace No. 3 (Table 1)**

**Conclusions.** The study investigated the influence of thermo-oxidative treatment conditions on the dimensional accuracy of hard-alloy products. It was established that the dispersion of dimensions after treatment is significantly affected by the nonuniformity of the temperature field in the furnace and by random temperature fluctuations during the oxidation process. Even small temperature differences within the working zone can lead to noticeable deviations in the thickness of the removed oxide layer and, consequently, in the final dimensions of the parts.

A theoretical model describing the relationship between temperature fluctuations and the variance of the removed layer during thermo-oxidative treatment was developed. The obtained analytical expressions make it possible to estimate the influence of technological parameters on dimensional deviations. Experimental studies performed on tungsten-cobalt hard-alloy samples confirmed the theoretical results and demonstrated a satisfactory agreement between calculated and measured values of dimensional dispersion.

The research also showed that the oxidation environment and processing parameters significantly influence dimensional accuracy. Thermo-oxidation improves the surface microrelief and can slightly reduce the dispersion of dimensions due to the smoothing of micro-irregularities on the surface. At the same time, increasing the oxidation time while reducing the oxidation rate and furnace temperature contributes to a decrease in dimensional variance.

Thus, optimization of the thermo-oxidative treatment regime, improvement of temperature control systems, and grouping of parts with similar initial dimensions are effective ways to increase the accuracy of dimensional processing of hard-alloy products. The obtained results can be used for improving technological processes in the manufacturing of precision hard-alloy components.

## References

1. Kaletnik, G.M. (2019). Perspektyvy pidvyshchennya enerhetychnoyi avtonomiyi pidpryemstv APK v ramkakh vykonannya enerhetychnoyi stratehiyi Ukrainy [Prospects for increasing the energy autonomy of agricultural enterprises in the framework of the energy strategy of Ukraine]. *Bulletin of Agrarian Science of the Black Sea Region*, 4, 90–98. DOI: [https://doi.org/10.31521/2313-092X/2019-4\(104\)-10](https://doi.org/10.31521/2313-092X/2019-4(104)-10). [in Ukrainian].
2. Kaletnik, G.M., Lutkovska, S.M. (2022). *Ekolohichna modernizatsiya ta orhanichne vyrobnytstvo v systemi ekolohichnoyi bezpeky: monohrafiya [Ecological modernization and organic production in the system of environmental security: monograph]*. Vinnytsia: VNAU. [in Ukrainian].
3. Kaletnik, H.M., Yaropud, V.M. (2021). Fyzyko-matematychna model' ventylyatsiynoyi systemy nahnitannya chystoho povitrya u tvarynnyts'kykh prymishchennyakh [Physico-mathematical model of the ventilation system for injecting clean air in livestock premises]. *Tekhnika, enerhetyka, transport APK*, 3 (114), 4–15. DOI: <https://doi.org/10.37128/2520-6168-2021-3-1>. [in Ukrainian].
4. Kaletnik, G.M., Yaropud, V.M. (2021). Theoretical studies of pneumatic losses of the air heat exchanger of the indirect-evaporative type of livestock premises [Teoretychni doslidzhennya pnevmovtrat povitryanoho teploobminnyka pobichno-vyparnoho typu tvarynnyts'kykh prymishchen']. *Machinery & Energetics*, 12 (4), 35–41. DOI: <http://dx.doi.org/10.31548/machenergy2021.04.03>
5. [in Ukrainian].
5. Kaletnik, G.M., Yaropud, V.M. (2022). Simulation of the heat and mass transfer process of the indirect-evaporative type heat exchanger [Symulyatsiya protsesu teplomasoobminu teploobminnyka pobichno-vyparnoho typu]. *Tekhnika, enerhetyka, transport APK*, 1 (116), 4–15. [in Ukrainian].
6. Dolgikh, D.O. (2021). Analiz roboty ta klasyfikatsiya gruntovykh teploobminnykiv [Analysis of operation and classification of soil heat exchangers]. *Collection of scientific works of the Institute of Mechanization of Animal Husbandry of the National Academy of Sciences «Mechanization, environmentalization and conversion of bio-raw materials in animal husbandry»*, 1 (9), 56–63. [in Ukrainian].
7. Blazquez, C.S., Borge-Diez, D., Nieto, I.M., Martín, A.F., González-Aguilera, D. (2023). Multiparametric evaluation of electrical, biogas and natural gas geothermal source heat pumps. *Green Energy and Technology*. Springer, Cham. 103–122. DOI: [https://doi.org/10.1007/978-3-031-24524-4\\_4](https://doi.org/10.1007/978-3-031-24524-4_4).



[in English].

8. Shakil, Masum, Liangliang, Jiang. (2023). Technical performance comparison of horizontal and vertical ground-source heat pump systems. *Journal of GeoEnergy*. <https://doi.org/10.1155/2023/6106360> [in English].

9. Hałaj, E, Pająk, L, Papiernik, B. (2020). Finite element modeling of geothermal source of heat pump in long-term operation. *Energies*, 13 (6), 1341. DOI: <https://doi.org/10.3390/en1306134>. [in English].

10. Neuberger, P, Adamovský, R, Šedová, M. (2014). Temperatures and heat flows in a soil enclosing a slinky horizontal heat exchanger. *Energies*, 7 (2), 972–987. DOI: <https://doi.org/10.3390/en7020972>. [in English].

11. Kovyazin, O.S., Dolgikh, D.O. (2013). Obgruntuvannya konstruktsiyi gruntovoho teploobminnyka [Ground heat exchanger construction justification]. *Bulletin of KhNUTSG named after P. Vasylenko*, 132, 167. [in Ukrainian].

12. Kovyazin, O.S. (2018). Obgruntuvannya diametra obsadnoyi truby gruntovoho teploobminnyka ta podachi povitrya v n'oho [Justification of the diameter of the casing pipe of the soil heat exchanger and air supply to it]. *Bulletin of the National Technical University «KhPI». Series: Energy and heat engineering processes and equipment*, 12 (1288). DOI: 10.20998/2078-774X.2018.12.13. [in Ukrainian].

13. Zhengxuan Liu, Mingjing Xie, Yuekuan Zhou, Yingdong He, Lei Zhang, Guoqiang Zhang, Dachuan Chen. (2023). A state-of-the-art review on shallow geothermal ventilation systems with thermal performance enhancement system classifications, advanced technologies and applications. *Energy and Built Environment*, 4 (2), 148–168. DOI: <https://doi.org/10.1016/j.enbenv.2021.10.003>. [in English].

14. Sydoruk, B., Naumchuk, O., Mazurek, P. (2021). Modeling of Joint Operation of a Ground Soil Heat Exchanger and a Thermal Pump Evaporator. *Journal of Ecological Engineering*, 22 (2), 256–261. DOI: <https://doi.org/10.12911/22998993/131177>. [in English].

15. Jalaluddin Jalaluddin, Miyara Akio, Tsubaki Koutaro, Yoshida Kentaro (2010). Thermal performances of three types of ground heat exchangers in short-time period of operation. *International Refrigeration and Air Conditioning Conference*. P. 1123. <http://docs.lib.purdue.edu/iracc/1123> [in English].

16. Aliyev, E.B., Yaropud, V.M. (2015). Porivnyal'nyy analiz rezul'tativ teoretychnykh y eksperymental'nykh doslidzhen' protsesu funktsionuvannya teploutylizatora dlya tvarynnyts'kykh prymishchen [Comparative analysis of the results of theoretical and

experimental studies of the process of functioning of the heat exchanger for livestock premises]. *Design, production and operation of agricultural machines*, 4 (II), 120–124. [in Ukrainian].

17. Kaletnik, H.M., Yaropud, V.M. (2022). Rezul'taty chysel'noho modelyuvannya heothermal'noho okhologzhennya u ventilyatsiyniy systemi tvarynnyts'kykh prymishchen [Results of numerical modeling of geothermal cooling in the ventilation system of livestock premises]. *Vibrations in engineering and technology*, 3 (106), 5–12. DOI: <https://doi.org/10.37128/2306-8744-2022-3-1>. [in Ukrainian].

Kaletnik, G.M., Yaropud, V.M. (2023). Eksperymental'ni doslidzhen'ya efektyvnosti funktsionuvannya system zabezpechennya mikroklimatu vid'yemnoho tysku v tvarynnyts'kykh prymishchennyakh [Experimental studies of the effectiveness of systems for providing negative pressure microclimate in livestock premises]. *Design, production and operation of agricultural machines*, 53, 66–84. DOI: <https://doi.org/10.32515/2414-3820.2023.53.66-84>. [in Ukrainian].

#### ОЦІНКА ТОЧНОСТІ ТЕХНОЛОГІЧНОЇ ОБРОБКИ ВИРОБІВ З КАРБІДУ МЕТОДОМ ТЕРМІЧНОГО ОКСИДУВАННЯ З МЕТОЮ ПІДВИЩЕННЯ МІЦНОСТІ

У статті досліджено точність розмірозміцнювальної термоокиснювальної обробки твердосплавних виробів. Проаналізовано, як нерівномірність температурного поля в печі та температурні коливання під час обробки впливають на розсіювання розмірів деталей із вольфрамкобальтового сплаву. На основі статистичних методів і теорії випадкових процесів розроблено теоретичну модель, що описує зв'язок між відхиленнями розмірів до і після термоокиснення.

Аналітично оцінено вплив сталих температурних градієнтів і випадкових температурних флуктуацій на глибину окиснення та точність розмірів. Визначено кореляційні функції температурних змін у печах із різними системами керування та встановлено їхній вплив на дисперсію шару матеріалу, що видаляється під час термоокиснювальної обробки. Експериментальні дослідження на твердосплавних зразках підтверджують теоретичні залежності та показують, що навіть незначні температурні перепади в печі можуть призводити до помітних відхилень розмірів.

У роботі також розглянуто вплив умов окиснення (у повітряному та кисневому середовищі) на розмірну точність



твердосплавних елементів, що застосовуються в камерах високого тиску. Результати показали, що термоокислювальна обробка покращує мікрорельєф поверхні та може дещо зменшувати розсіювання розмірів. Встановлено, що збільшення тривалості окиснення за одночасного зниження швидкості окиснення та температури печі дає змогу істотно зменшити дисперсію розмірів і підвищити точність обробки.

Отримані результати дають підстави сформулювати рекомендації щодо оптимізації

параметрів термоокислювальної обробки та підвищення розмірної точності під час виготовлення твердосплавних компонентів.

**Ключові слова:** термоокислювальна обробка, тверді сплави, вольфрамо-кобальтові сплави, точність розмірів, коливання температури, кінетика окиснення, дисперсія розмірів, неоднорідність температурного поля, глибина окиснення, мікрорельєф поверхні.

#### **Відомості про авторів**

**Paulikienė Simona**, Department of Mechanical, Energy and Biotechnology Engineering, Faculty of Engineering, Agriculture Academy, Vytautas Magnus University, Studentų Str. 15A, Akademija, LT-53362 Akademija, Kaunas district, Kaunas, Lithuania; [simona.paulikiene1@vdu.lt](mailto:simona.paulikiene1@vdu.lt), <https://orcid.org/0000-0002-8803-2941>

**Glowacki Szymon**, Department of Fundamentals of Engineering and Power Engineering, Institute of Mechanical Engineering, Warsaw University of Life Sciences (SGGW), 02-787 Warsaw, Poland; [szymon\\_glowacki@sggw.edu.pl](mailto:szymon_glowacki@sggw.edu.pl) <https://orcid.org/0000-0002-0373-6633>

Стаття надійшла 30.01.2026

Стаття прийнята 10.02.2026

Опубліковано 17.04.2026